

**REPUBLIC OF SOUTH-AFRICA
DEPARTMENT OF MINERALS AND ENERGY
EXAMINATION FOR THE MINE SURVEYOR'S CERTIFICATE OF COMPETENCY**

DATE: 15 April 2004 (Thursday)
TIME: 12:30 – 15:30 (3 Hours)

TOTAL MARKS: 100
TO PASS: 50

MINING ECONOMICS II

NOTE:

1. Any pocket calculator may be used and intermediate results need not be shown. The make and model number of the calculator used must be noted on the front cover of answer book.
2. Tables that may be used are attached hereto.
3. Graph-, Probability-, and Log paper will be supplied if required.
4. Your examination number must be written on all graph paper and loose sheets that are handed in with your examination script.

QUESTION 1

2, 4, 5, 3, 3, 9, 6, 3, 4, 2

- a) For the above set of samples, calculate the following without the use of the statistical functions of your calculator and showing logical steps:
 - i) The expected value of the population mean.
 - ii) The sample standard deviation.
 - iii) The estimator of the population standard deviation.
- b) Draw a histogram of the sample population.
- c) What is the mode of the data?
- d) What is the median of the data?
- e) To which population does the data belong?

[10 marks]

QUESTION 2

Given the following g/t gold values:

Interval g/t	Frequency
0-10	14
10-20	33
20-30	24
30-40	13
40-50	10
50-60	6
60-70	3
70-80	2
80-90	4

- As a test for normality, plot the data on the probability paper provided.
- What distribution are we dealing with?

[8 Marks]

QUESTION 3

The table below shows the monthly tons milled and related cost at a gold mine.

	Tons milled	R (million)
Jan	78 000	26,91
Feb	73 000	24,75
March	63 500	23,63
April	97 000	28,04
May	93 000	26,50
June	89 000	26,48

The pay limit of 8,9g/t for the mine was calculated using 82 000 tons milled monthly at a cost of $R26 \times 10^6$

Due to unavailability of stope face it is however not possible to produce the 82 000 tons planned. It is envisaged that only 75 000 tons will be produced per month and that this reduction will not affect fixed cost.

Calculate the effect of the change of scale of operations on the pay limit.

[10 Marks]

QUESTION 4

- a) What is the present value of the following profits earned over a period of 6 years at 5% compound interest for the first 3 years and 7,5% for the next 3 years?

Year	Profit Rand
1	120 000
2	60 000
3	20 000
4	-30 000
5	20 000
6	15 000

(8)

- b) The proposed raise bore ore pass costing R438 000 is expected to reduce transport cost of a mine by 4c per ton. If the mine has a life of 10 years and a milling rate of 1 850 000 tons per annum, is the proposed raise bore hole worthwhile? (7)

[15 Marks]

QUESTION 5

Based on 100 ten ton samples, ore at an opencast base metal mine is normally distributed with a mean grade of 2,2 % and standard deviation of 1,8% and the total ore reserve is 195 million tons. The bench height and drilling pattern however allows a smallest mining unit of 1 050 tons.

Draw grade tonnage curves from 1,5% to 2,6% metal for the mining unit.

[20Marks]

QUESTION 6

In the tabulation below, X are the original estimated block values in g/t and Y the actual values of the blocks as they were mined.

Y	6,1	6,2	6,4	6,4	6,6	6,8	6,9	6,9	7,2	7,4	7,6	7,7	7,7	7,8
X	6,2	6,8	6,6	7,0	6,2	7,7	6,6	7,2	8,0	6,9	7,7	8,2	7,3	6,7
Y	8,0	8,1	8,2	8,3	8,4	8,5	8,8	8,8	8,9	10,2	10,4			
X	8,0	8,6	7,4	7,0	8,2	7,8	8,7	8,1	8,5	8,4	9,0			

Calculate the:

- Correlation coefficient.
- Regression line.
- Covariance.
- Test whether $\rho=0$ at the 0,05 level of significance.
- What is the probability distribution of the actual value of a block whose estimated value is 8,0g/t.
- Calculate the effective pay limit by which a sample may be judged if the mining pay limit is 7,0g/t.

[20Marks]

QUESTION 7

- (a) Make an annotated sketch of the following semi-variogram and briefly discuss the various features of the graph.

$$\gamma(0) = 1,3$$

$$\gamma(h) = C_0 + C(3h/2a - h^3/2a^3) \quad \text{when } 0 < h < a$$

$$\gamma(h) = C_0 + C \quad \text{when } h > a$$

$$C = 3,2 \quad a = 30.$$

- (b) Describe:
- i) Stationarity
 - ii) Intrinsic
 - iii) Isotropic
 - iv) Periodicity

[17 Marks]

[TOTAL 100 Marks]

SOME USEFUL FORMULAE

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^N (x_i - u)^2$$

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^N x_i^2 - u^2$$

$$s^2 = \frac{1}{n-1} \sum (x - \bar{x})^2$$

$$\gamma(h) = C \left(\frac{3h}{2a} - \frac{h^3}{2a^3} \right)$$

$$s^2 = \frac{1}{n-1} [\sum x^2 - n\bar{x}^2]$$

$$\gamma(h) = C(1 - \exp(\frac{-h}{a}))$$

$$T = \frac{\bar{x} - \mu}{s\sqrt{n}}$$

$$T = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

$$P = PV \frac{i}{1 - (1+i)^n}$$

$$PV = A/(1+r)^n$$

$$PV = P(1 - (1+i)^{-n})/i$$

$$PV = \frac{P[(1+r)^n - 1]}{r(1+r)^n}$$

$$\theta = \frac{Mx^2 - x_p x_{1-p}}{x_p + x_{1-p} - 2Mx}$$

$$b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}$$

$$2\gamma(h) = \frac{1}{n} \sum [g(x) - g(x+h)]$$

$$r = \frac{\sum xy - \sum x \sum y / n}{[\sum x^2 - (\sum x)^2 / n][\sum y^2 - (\sum y)^2 / n]}$$

$$\sigma^c = \hat{s}_y \sqrt{1-r^2}$$

99.99 99.9 99.8 99 98 95 90 80 70 60 50 40 30 20 10 5 2 1 0.3 0.2 0.1 0.05 0.01

0.01 0.05 0.1 0.2 0.3 1 2 5 10 20 30 40 50 60 70 80 90 95 98 99 99.8 99.9 99.99

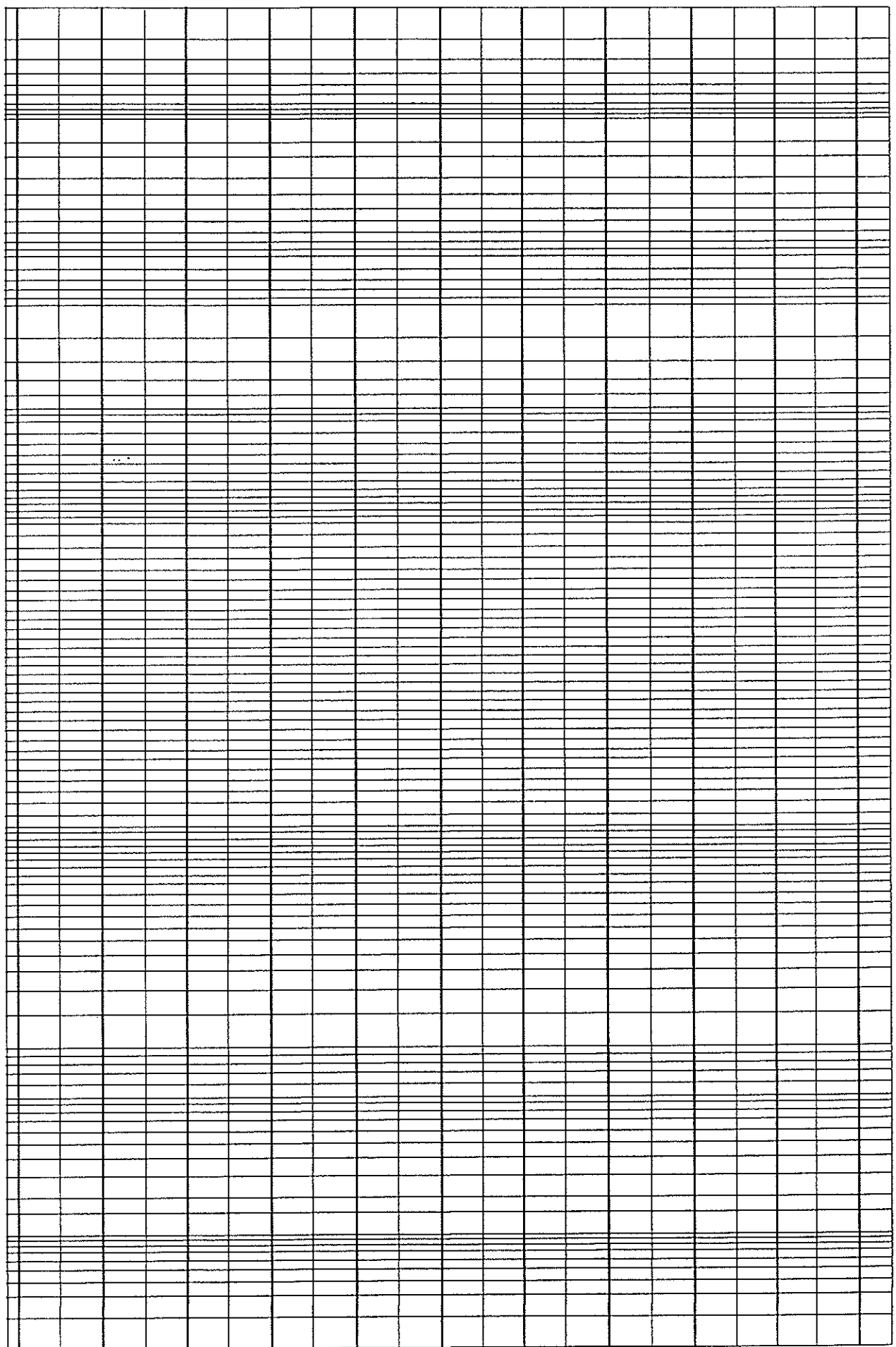


TABLE 5

TABLE FOR THE COMPUTATION OF TONNAGE AND GRADE CUT-OFF FOR THE NORMAL DISTRIBUTION				
Cut-off below the mean		Reduced cut-off = Z PV=MV+ $\omega\sigma$	Cut-off above the mean	
Tonnage proportion	ω - factor		ω - factor	Tonnage proportion
50.00	0.798	0.00	0.798	50.00
51.99	0.766	0.05	0.830	48.01
53.98	0.735	0.10	0.863	46.02
55.96	0.705	0.15	0.896	44.04
57.93	0.675	0.20	0.929	42.07
59.87	0.645	0.25	0.964	40.13
61.79	0.617	0.30	0.998	38.21
63.68	0.589	0.35	1.034	36.32
65.54	0.562	0.40	1.069	34.45
67.36	0.535	0.45	1.106	32.64
69.15	0.509	0.50	1.142	30.85
70.88	0.484	0.55	1.180	29.12
72.57	0.459	0.60	1.217	27.43
74.22	0.435	0.65	1.256	25.78
75.80	0.411	0.70	1.295	24.20
77.34	0.389	0.75	1.334	22.66
78.81	0.367	0.80	1.375	21.19
80.23	0.346	0.85	1.415	19.77
81.59	0.326	0.90	1.457	18.41
82.89	0.306	0.95	1.499	17.11
84.13	0.287	1.00	1.542	15.87
85.31	0.269	1.05	1.586	14.69
86.43	0.251	1.10	1.631	13.57
87.49	0.235	1.15	1.677	12.51
88.49	0.219	1.20	1.724	11.51
89.44	0.204	1.25	1.772	10.56
90.32	0.189	1.30	1.821	9.63
91.15	0.175	1.35	1.872	8.35
91.92	0.162	1.40	1.923	8.08
92.65	0.150	1.45	1.977	7.35
93.32	0.133	1.50	2.033	6.68
93.94	0.127	1.55	2.098	6.06
94.52	0.117	1.60	2.147	5.48
95.05	0.107	1.65	2.208	4.95
95.34	0.098	1.70	2.270	4.46
95.99	0.090	1.75	2.335	4.01
96.41	0.082	1.80	2.403	3.59
96.78	0.074	1.85	2.473	3.22
97.13	0.067	1.90	2.546	2.87
97.44	0.061	1.95	2.622	2.56
97.72	0.055	2.00	2.701	2.28
97.98	0.050	2.05	2.784	2.02
98.21	0.045	2.10	2.870	1.79
98.42	0.040	2.15	2.961	1.58
98.61	0.036	2.20	3.055	1.39
98.78	0.032	2.25	3.155	1.22

LOWER 95%

Factor $b_{0.95}(v, n)$ for estimation of one sided lower 95% confidence limits of the mean of a lognormal population

\sqrt{v}	5	10	15	20	50	100	1000
0.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.02	0.8978	0.9333	0.9458	0.9540	0.9697	0.9782	0.9927
0.04	0.8569	0.9071	0.9246	0.9344	0.9573	0.9692	0.9895
0.05	0.8302	0.8874	0.9079	0.9200	0.9478	0.9622	0.9872
0.08	0.8070	0.8708	0.8943	0.9077	0.9398	0.9564	0.9852
0.10	0.7870	0.8563	0.8821	0.8972	0.9328	0.9512	0.9833
0.12	0.7693	0.8439	0.8716	0.8878	0.9264	0.9464	0.9817
0.14	0.7535	0.8323	0.8617	0.8790	0.9204	0.9420	0.9801
0.16	0.7389	0.8216	0.8527	0.8709	0.9149	0.9380	0.9787
0.18	0.7255	0.8116	0.8442	0.8632	0.9097	0.9341	0.9773
0.20	0.7129	0.8023	0.8360	0.8558	0.9048	0.9304	0.9760
0.30	0.6605	0.7618	0.8008	0.8243	0.8828	0.9139	0.9701
0.40	0.6187	0.7284	0.7717	0.7981	0.8639	0.8996	0.9648
0.50	0.5838	0.6995	0.7462	0.7744	0.8470	0.8867	0.9600
0.60	0.5538	0.6739	0.7270	0.7534	0.8313	0.8741	0.9554
0.70	0.5277	0.6508	0.7020	0.7338	0.8168	0.8632	0.9511
0.80	0.5044	0.6297	0.6825	0.7156	0.8030	0.8525	0.9470
0.90	0.4836	0.6103	0.6646	0.6987	0.7899	0.8421	0.9429
1.00	0.4650	0.5923	0.6476	0.6826	0.7774	0.8322	0.9389
1.10	0.4481	0.5756	0.6317	0.6674	0.7654	0.8226	0.9351
1.20	0.4328	0.5599	0.6165	0.6530	0.7538	0.8133	0.9313
1.30	0.4189	0.5452	0.6023	0.6393	0.7426	0.8042	0.9276
1.40	0.4062	0.5315	0.5888	0.6262	0.7318	0.7954	0.9240
1.50	0.3946	0.5186	0.5760	0.6137	0.7214	0.7868	0.9203
1.60	0.3840	0.5065	0.5637	0.6018	0.7112	0.7784	0.9168
1.70	0.3743	0.4950	0.5521	0.5904	0.7014	0.7702	0.9133
1.80	0.3655	0.4842	0.5410	0.5794	0.6918	0.7622	0.9098
1.90	0.3574	0.4740	0.5305	0.5688	0.6825	0.7544	0.9064
2.00	0.3501	0.4644	0.5203	0.5587	0.6734	0.7466	0.9030
2.10	0.3433	0.4552	0.5106	0.5489	0.6646	0.7391	0.8996
2.20	0.3372	0.4466	0.5014	0.5395	0.6560	0.7317	0.8962
2.30	0.3316	0.4385	0.4925	0.5304	0.6476	0.7245	0.8929
2.40	0.3266	0.4308	0.4840	0.5217	0.6394	0.7173	0.8896
2.50	0.3220	0.4234	0.4759	0.5133	0.6314	0.7104	0.8864
2.60	0.3179	0.4166	0.4681	0.5044	0.6236	0.7035	0.8831
2.70	0.3142	0.4100	0.4606	0.4974	0.6160	0.6967	0.8799
2.80	0.3110	0.4039	0.4535	0.4899	0.6085	0.6901	0.8767
2.90	0.3081	0.3981	0.4467	0.4826	0.6012	0.6836	0.8736
3.00	0.3055	0.3926	0.4401	0.4756	0.5941	0.6772	0.8704

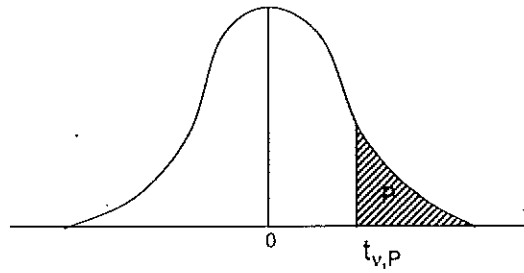
UPPER 95%

Factor $b_{0.95}(v, n)$ for estimation of one sided upper 95% confidence limits of the mean of a lognormal population

\sqrt{v}	5	10	15	20	50	100	1000
0.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.02	1.241	1.117	1.084	1.067	1.038	1.026	1.007
0.04	1.362	1.171	1.122	1.099	1.055	1.037	1.011
0.05	1.466	1.216	1.154	1.124	1.069	1.046	1.013
0.08	1.561	1.256	1.181	1.146	1.080	1.053	1.015
0.10	1.652	1.293	1.207	1.166	1.091	1.060	1.017
0.12	1.740	1.327	1.230	1.184	1.100	1.066	1.019
0.14	1.827	1.361	1.253	1.202	1.109	1.072	1.020
0.16	1.914	1.393	1.274	1.219	1.118	1.078	1.022
0.18	1.999	1.425	1.295	1.236	1.126	1.084	1.023
0.20	2.087	1.455	1.316	1.252	1.135	1.089	1.025
0.30	2.532	1.606	1.415	1.328	1.172	1.113	1.031
0.40	3.019	1.756	1.509	1.399	1.207	1.135	1.037
0.50	3.563	1.910	1.603	1.470	1.240	1.156	1.042
0.60	4.176	2.070	1.682	1.541	1.273	1.175	1.047
0.70	4.870	2.237	1.798	1.614	1.306	1.196	1.052
0.80	5.663	2.415	1.901	1.688	1.338	1.215	1.057
0.90	6.570	2.604	2.006	1.763	1.371	1.235	1.062
1.00	7.605	2.805	2.117	1.842	1.404	1.254	1.067
1.10	8.795	3.019	2.233	1.924	1.437	1.274	1.071
1.20	10.155	3.250	2.355	2.008	1.471	1.294	1.076
1.30	11.718	3.497	2.483	2.096	1.506	1.314	1.080
1.40	13.513	3.761	2.617	2.187	1.540	1.334	1.085
1.50	15.569	4.045	2.758	2.282	1.576	1.354	1.089
1.60	17.928	4.351	2.907	2.380	1.613	1.374	1.094
1.70	20.639	4.680	3.064	2.484	1.650	1.395	1.098
1.80	23.749	5.034	3.229	2.592	1.688	1.416	1.103
1.90	27.318	5.414	3.403	2.704	1.728	1.438	1.107
2.00	31.398	5.825	3.588	2.822	1.767	1.459	1.112
2.10	36.079	6.268	3.783	2.945	1.808	1.481	1.116
2.20	41.444	6.745	3.989	3.074	1.850	1.504	1.121
2.30	47.586	7.260	4.208	3.209	1.893	1.526	1.125
2.40	54.611	7.815	4.438	3.351	1.937	1.549	1.130
2.50	62.661	8.415	4.683	3.498	1.982	1.572	1.134
2.60	71.861	9.061	4.941	3.670	2.029	1.596	1.139
2.70	82.366	9.759	5.214	3.816	2.076	1.620	1.144
2.80	94.377	10.512	5.504	3.986	2.125	1.645	1.148
2.90	108.115	11.326	5.811	4.164	2.175	1.670	1.153
3.00	123.750	12.205	6.137	4.351	2.226	1.695	1.158

TABLE 3

The t-Distribution :
 Upper Probability Points
 $P = P(t \geq t_{v,P}) = P(t \leq -t_{v,P})$
 with $t_{v,P} = -t_{v,1-P}$ so that
 $P(|t| \geq t_{v,P}) = 2P, \quad t_{v,P} > 0.$



Entries in the table are the values $t_{v,P}$ of the t-distribution for various degrees of freedom v and one tailed probabilities P .

$v \backslash P$	0.25	0.10	0.05	0.025	0.01	0.005
1	1.000	3.078	6.314	12.706	31.821	63.657
2	0.816	1.886	2.920	4.303	6.965	9.925
3	0.765	1.638	2.353	3.182	4.541	5.841
4	0.741	1.533	2.132	2.776	3.747	4.604
5	0.727	1.476	2.015	2.571	3.365	4.032
6	0.718	1.440	1.943	2.447	3.143	3.707
7	0.711	1.415	1.895	2.365	2.998	3.499
8	0.706	1.397	1.860	2.306	2.896	3.355
9	0.703	1.383	1.833	2.262	2.821	3.250
10	0.700	1.372	1.812	2.228	2.764	3.169
11	0.697	1.363	1.796	2.201	2.718	3.106
12	0.695	1.356	1.782	2.179	2.681	3.055
13	0.694	1.350	1.771	2.160	2.650	3.012
14	0.692	1.345	1.761	2.145	2.624	2.977
15	0.691	1.341	1.753	2.131	2.602	2.947
16	0.690	1.337	1.746	2.120	2.583	2.921
17	0.689	1.333	1.740	2.110	2.567	2.898
18	0.688	1.330	1.734	2.101	2.552	2.878
19	0.688	1.328	1.729	2.093	2.539	2.861
20	0.687	1.325	1.725	2.086	2.528	2.845
21	0.686	1.323	1.721	2.080	2.518	2.831
22	0.686	1.321	1.717	2.074	2.508	2.819
23	0.685	1.319	1.714	2.069	2.500	2.807
24	0.685	1.318	1.711	2.064	2.492	2.797
25	0.684	1.316	1.708	2.060	2.485	2.787
26	0.684	1.315	1.706	2.056	2.479	2.779
27	0.684	1.314	1.703	2.052	2.473	2.771
28	0.683	1.313	1.701	2.048	2.467	2.763
29	0.683	1.311	1.699	2.045	2.462	2.756
30	0.683	1.310	1.697	2.042	2.457	2.750
35	0.682	1.306	1.690	2.030	2.438	2.724
40	0.681	1.303	1.684	2.021	2.423	2.704
60	0.679	1.296	1.671	2.000	2.390	2.660
100	0.677	1.290	1.660	1.984	2.364	2.626
α	0.675	1.282	1.645	1.960	2.326	2.576

TABLE 4

FACTOR $\gamma_{\eta}(v)$ FOR ESTIMATION OF MEAN OF LOGNORMAL POPULATION

$v \backslash \eta$	2	3	4	5	6	7	8	9	10	11	12
0.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.02	1.010	1.010	1.010	1.010	1.010	1.010	1.010	1.010	1.010	1.010	1.010
0.04	1.020	1.020	1.020	1.020	1.020	1.020	1.020	1.020	1.020	1.020	1.020
0.06	1.030	1.030	1.030	1.030	1.030	1.030	1.030	1.030	1.030	1.030	1.030
0.08	1.040	1.040	1.040	1.040	1.040	1.041	1.041	1.041	1.041	1.041	1.041
0.10	1.050	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051
0.12	1.061	1.061	1.061	1.061	1.061	1.061	1.061	1.061	1.061	1.062	1.062
0.14	1.071	1.071	1.071	1.072	1.072	1.072	1.072	1.072	1.072	1.072	1.072
0.16	1.081	1.082	1.082	1.082	1.082	1.082	1.082	1.083	1.083	1.083	1.083
0.18	1.091	1.092	1.092	1.093	1.093	1.093	1.093	1.093	1.093	1.093	1.094
0.20	1.102	1.102	1.103	1.103	1.104	1.104	1.104	1.104	1.104	1.104	1.104
0.3	1.154	1.156	1.157	1.158	1.158	1.159	1.159	1.159	1.160	1.160	1.160
0.4	1.207	1.210	1.212	1.214	1.215	1.216	1.216	1.217	1.217	1.217	1.218
0.5	1.260	1.266	1.269	1.272	1.273	1.275	1.276	1.276	1.277	1.278	1.278
0.6	1.315	1.323	1.328	1.332	1.334	1.336	1.337	1.338	1.339	1.340	1.341
0.7	1.371	1.382	1.389	1.393	1.397	1.399	1.401	1.403	1.404	1.406	1.406
0.8	1.427	1.442	1.451	1.457	1.462	1.465	1.468	1.470	1.472	1.473	1.475
0.9	1.485	1.503	1.515	1.523	1.529	1.533	1.537	1.540	1.542	1.544	1.546
1.0	1.543	1.566	1.580	1.591	1.598	1.604	1.608	1.612	1.615	1.618	1.620
1.1	1.602	1.630	1.648	1.661	1.670	1.677	1.682	1.687	1.691	1.694	1.697
1.2	1.662	1.696	1.718	1.733	1.744	1.752	1.759	1.765	1.770	1.774	1.777
1.3	1.724	1.764	1.789	1.807	1.820	1.831	1.839	1.846	1.851	1.856	1.860
1.4	1.786	1.832	1.862	1.884	1.900	1.912	1.922	1.930	1.936	1.942	1.947
1.5	1.848	1.903	1.938	1.963	1.981	1.996	2.007	2.017	2.025	2.032	2.037
1.6	1.912	1.975	2.015	2.044	2.066	2.082	2.096	2.107	2.116	2.124	2.131
1.7	1.977	2.049	2.095	2.128	2.153	2.172	2.188	2.201	2.212	2.221	2.229
1.8	2.043	2.124	2.177	2.214	2.243	2.265	2.283	2.298	2.310	2.321	2.330
1.9	2.110	2.201	2.260	2.303	2.336	2.361	2.382	2.399	2.413	2.425	2.436
2.0	2.178	2.280	2.347	2.395	2.431	2.460	2.484	2.503	2.519	2.533	2.545
2.1	2.247	2.360	2.435	2.489	2.530	2.563	2.589	2.611	2.630	2.645	2.659
2.2	2.317	2.442	2.526	2.586	2.632	2.669	2.698	2.723	2.744	2.762	2.778
2.3	2.388	2.526	2.618	2.686	2.737	2.778	2.811	2.839	2.863	2.883	2.900
2.4	2.460	2.612	2.714	2.788	2.846	2.891	2.928	2.959	2.986	3.008	3.028
2.5	2.533	2.699	2.812	2.894	2.957	3.008	3.049	3.084	3.113	3.138	3.160
2.6	2.607	2.789	2.912	3.003	3.073	3.128	3.174	3.213	3.245	3.274	3.298
2.7	2.682	2.880	3.015	3.114	3.191	3.253	3.304	3.346	3.382	3.414	3.441
2.8	2.759	2.973	3.120	3.229	3.314	3.382	3.437	3.484	3.524	3.559	3.589
2.9	2.836	3.068	3.228	3.347	3.440	3.514	3.576	3.627	3.671	3.710	3.743
3.0	2.914	3.166	3.339	3.469	3.570	3.651	3.718	3.775	3.824	3.866	3.902
3.1	2.994	3.265	3.453	3.593	3.703	3.792	3.866	3.928	3.981	4.028	4.068
3.2	3.075	3.366	3.569	3.721	3.841	3.938	4.018	4.086	4.145	4.195	4.240
3.3	3.157	3.469	3.688	3.853	3.983	4.088	4.176	4.250	4.314	4.369	4.418
3.4	3.240	3.574	3.810	3.988	4.129	4.243	4.338	4.419	4.489	4.549	4.603
3.5	3.324	3.682	3.935	4.127	4.279	4.403	4.506	4.594	4.670	4.736	4.794
3.6	3.409	3.792	4.063	4.270	4.434	4.568	4.680	4.775	4.858	4.929	4.993
3.7	3.496	3.903	4.194	4.416	4.593	4.738	4.859	4.962	5.052	5.130	5.198
3.8	3.583	4.017	4.329	4.567	4.757	4.913	5.044	5.156	5.252	5.337	5.412
3.9	3.672	4.134	4.466	4.721	4.925	5.093	5.234	5.355	5.460	5.552	5.633
4.0	3.762	4.252	4.607	4.880	5.099	5.279	5.431	5.562	5.675	5.774	5.862
4.1	3.853	4.373	4.751	5.042	5.277	5.471	5.634	5.775	5.897	6.004	6.099
4.2	3.946	4.496	4.898	5.209	5.460	5.668	5.844	5.995	6.127	6.242	6.345
4.3	4.040	4.622	5.049	5.380	5.649	5.872	6.060	6.223	6.364	6.489	6.599
4.4	4.135	4.750	5.203	5.556	5.843	6.081	6.283	6.458	6.610	6.744	6.863
4.5	4.231	4.881	5.361	5.736	6.042	6.297	6.513	6.700	6.863	7.008	7.136
4.6	4.328	5.014	5.522	5.921	6.247	6.519	6.750	6.950	7.126	7.281	7.419
4.7	4.427	5.149	5.687	6.111	6.457	6.747	6.995	7.209	7.397	7.563	7.711
4.8	4.527	5.288	5.856	6.305	6.674	6.983	7.247	7.476	7.677	7.855	8.014
4.9	4.629	5.428	6.029	6.505	6.896	7.225	7.507	7.751	7.966	8.157	8.328
5.0	4.732	5.572	6.205	6.709	7.124	7.474	7.774	8.036	8.265	8.470	8.652
5.1	4.836	5.718	6.386	6.919	7.359	7.731	8.050	8.329	8.574	8.792	8.988
5.2	4.941	5.866	6.570	7.134	7.600	7.995	8.335	8.631	8.893	9.126	9.335
5.3	5.048	6.018	6.759	7.354	7.847	8.266	8.628	8.944	9.222	9.471	9.695
5.4	5.156	6.172	6.951	7.579	8.102	8.546	8.930	9.265	9.563	9.828	10.07
5.5	5.266	6.329	7.148	7.811	8.363	8.833	9.240	9.598	9.914	10.20	10.45
5.6	5.376	6.489	7.350	8.048	8.631	9.129	9.561	9.940	10.28	10.58	10.85
5.7	5.489	6.652	7.555	8.290	8.906	9.433	9.890	10.29	10.65	10.97	11.26
5.8	5.603	6.818	7.766	8.539	9.188	9.745	10.23	10.66	11.04	11.38	11.68
5.9	5.718	6.987	7.980	8.794	9.478	10.07	10.58	11.03	11.44	11.80	
6.0	5.834	7.159	8.200	9.054	9.776	10.84	10.84	11.42	11.85		

