REPUBLIC OF SOUTH AFRICA
DEPARTMENT OF MINERALS AND ENERGY
EXAMINATION FOR THE MINE SURVEYOR'S CERTIFICATE OF COMPETENCY

DATE: 20 April 2006 (Thursday) TOTAL MARKS: 100
TIME: 12:30 – 15:30 (3 Hours) TO PASS: 50

MINING ECONOMICS II

NOTE:

1. Any pocket calculator may be used and intermediate results need not be shown. The make and model number of the calculator used must be noted on the front cover of the answer book.
2. Tables that may be used are attached hereto.
3. Graph, Probability and Log paper will be supplied if required
4. Your examination number must be written on all graph paper and loose sheets that are handed in with your examination script.

Question 1

Explain what is meant by the following variogram characteristics:

a) Range or Zone of Influence (4)
b) Behaviour near the origin (briefly describe the various types) (8)
c) Anisotropy (list 2 types) (4)
d) Nested structures (2)

[18]

Question 2

When modeling a semi-variogram, briefly list the necessary steps that should be followed.

[7]

Question 3

The following are borehole values (in cm.g/t) that have been drilled on a gold mining property.

310 430 510 520 680 680 800 980 1280 1690 2650 5100 10800

The additive constant for the values is 480cm.g/t.
The mining property covered by the boreholes is 1450 hectares in extent.
The average dip of the reef for this area is estimated to be 32°.
Density of the ore is 2.75 t/m³
It is estimated that mining will take place at a mining In variance of 0.3.
Geological losses in the area is estimated at 10%

Further estimates are:

Surface sorting	10% @ 0.4 g/t
MCF	92%
Recovery factor	97%
Shortfall	8% of tons hoisted
Stope width	120 cm
Mining pay limit	8.0g/t

It is expected that an average of 180 000 tons per month will be milled

Calculate

a) The average recovery grade
b) The life of mine in years.

[20]

Question 4

Draw annotated sketches of the following variogram models:

a) Exponential model
b) Spherical model
c) Linear model
d) Parabolic model

[12]

Question 5

The following details, based on 180 two-ton samples, are available:

Mean Value	30.9% silver
Standard deviation	7.9% silver
Size of deposit	18 million tons of ore
Minimum quantity that can be transported	30 tons

a) Determine grade tonnage curves for cut-off values ranging from 27.5% to 34.5% silver.
b) Read from the graphs, the grade and payable tons for the cut-off values of 30% and 32% silver.

[20]

Question 6

Determine the semi-variance value (γ) for the following samples values taken in a reef drive at 2 metre intervals.

21 23 19 12 18 13

[5]

Question 7

A gold mine has an estimated life of 15 years. A capital project having a cost of R80 million would reduce working costs by R0.75 per ton and increase recovered grade by 0.05 g/t. The milling rate is 120 000 tons per month.
What gold price must prevail to give a return of 5% before tax?

[6]

Question 8

The table listed below shows the X values of a mineral together with its corresponding Y values

X	4	9	10	12	14	16	20	20
Y	10	14	14	16	18	14	20	19

Determine the following:

a) The regression line to estimate Y values from the X values
b) The correlation coefficient between the two variables
c) Standard deviations for the X, Y and Z and error distributions
d) The effective pay limit if the official pay limit is 10.2
e) The Z value for an X value of 11.5
f) The probability of the Z value above proving to be unpay
g) The 90% confidence limits for the regressed value

[12]

Total Marks [100]

LOWER 95%

Factor $b_{0.05}(v;n)$ for estimation of one sided lower 95% confidence limits of the mean of a lognormal population

$v\backslash n$	5	10	15	20	50	100	1000
0.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.02	0.8978	0.9333	0.9458	0.9540	0.9697	0.9782	0.9927
0.04	0.8589	0.9071	0.9246	0.9344	0.9573	0.9692	0.9895
0.06	0.8302	0.8874	0.9079	0.9200	0.9478	0.9622	0.9872
0.08	0.8070	0.8708	0.8943	0.9077	0.9398	0.9564	0.9852
0.10	0.7870	0.8563	0.8821	0.8972	0.9328	0.9512	0.9833
0.12	0.7693	0.8439	0.8716	0.8878	0.9264	0.9464	0.9817
0.14	0.7535	0.8323	0.8617	0.8790	0.9204	0.9420	0.9801
0.16	0.7389	0.8216	0.8527	0.8709	0.9149	0.9380	0.9787
0.18	0.7255	0.8116	0.8442	0.8632	0.9097	0.9341	0.9773
0.20	0.7129	0.8023	0.8360	0.8558	0.9048	0.9304	0.9760
0.30	0.6605	0.7618	0.8008	0.8243	0.8828	0.9139	0.9701
0.40	0.6187	0.7284	0.7717	0.7981	0.8639	0.8996	0.9648
0.50	0.5838	0.6995	0.7462	0.7744	0.8470	0.8867	0.9600
0.60	0.5538	0.6739	0.7270	0.7534	0.8313	0.8741	0.9554
0.70	0.5277	0.6508	0.7020	0.7338	0.8168	0.8632	0.9511
0.80	0.5044	0.6297	0.6825	0.7156	0.8030	0.8525	0.9470
0.90	0.4836	0.6103	0.6646	0.6987	0.7899	0.8421	0.9429
1.00	0.4650	0.5923	0.6476	0.6826	0.7774	0.8322	0.9389
1.10	0.4481	0.5756	0.6317	0.6674	0.7654	0.8226	0.9351
1.20	0.4328	0.5599	0.6165	0.6530	0.7538	0.8133	0.9313
1.30	0.4189	0.5452	0.6023	0.6393	0.7426	0.8042	0.9276
1.40	0.4062	0.5315	0.5888	0.6262	0.7318	0.7954	0.9240
1.50	0.3946	0.5186	0.5760	0.6137	0.7214	0.7868	0.9203
1.60	0.3840	0.5065	0.5637	0.6018	0.7112	0.7784	0.9168
1.70	0.3743	0.4950	0.5521	0.5904	0.7014	0.7702	0.9133
1.80	0.3655	0.4842	0.5410	0.5794	0.6918	0.7622	0.9098
1.90	0.3574	0.4740	0.5305	0.5688	0.6825	0.7544	0.9064
2.00	0.3501	0.4644	0.5203	0.5587	0.6734	0.7466	0.9030
2.10	0.3433	0.4552	0.5106	0.5489	0.6646	0.7391	0.8996
2.20	0.3372	0.4466	0.5014	0.5395	0.6560	0.7317	0.8962
2.30	0.3316	0.4385	0.4925	0.5304	0.6476	0.7245	0.8929
2.40	0.3266	0.4308	0.4840	0.5217	0.6394	0.7173	0.8896
2.50	0.3220	0.4234	0.4759	0.5133	0.6314	0.7104	0.8864
2.60	0.3179	0.4166	0.4681	0.5044	0.6236	0.7035	0.8831
2.70	0.3142	0.4100	0.4606	0.4974	0.6160	0.6967	0.8799
2.80	0.3110	0.4039	0.4535	0.4899	0.6085	0.6901	0.8767
2.90	0.3081	0.3981	0.4467	0.482	0.6[illegible]	0.6836	0.8736
3.00	0.3055	0.3926	0.4401	0.4756	0.5[illegible]	0.6772	0.8704

UPPER 95%

Factor $b_{0.95}(v;n)$ for estimation of one sided upper 95% confidence limits of the mean of a lognormal population

$v\backslash n$	5	10	15	20	50	100	1000
0.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.02	1.241	1.117	1.084	1.067	1.038	1.026	1.007
0.04	1.362	1.171	1.122	1.099	1.055	1.037	1.011
0.06	1.466	1.216	1.154	1.124	1.069	1.046	1.013
0.08	1.561	1.256	1.181	1.146	1.080	1.053	1.015
0.10	1.652	1.293	1.207	1.166	1.091	1.060	1.017
0.12	1.740	1.327	1.230	1.184	1.100	1.066	1.019
0.14	1.827	1.361	1.253	1.202	1.109	1.072	1.020
0.16	1.914	1.393	1.274	1.219	1.118	1.078	1.022
0.18	1.999	1.425	1.295	1.236	1.126	1.084	1.023
0.20	2.087	1.455	1.316	1.252	1.135	1.089	1.025
0.30	2.532	1.605	1.415	1.328	1.172	1.113	1.031
0.40	3.019	1.755	1.509	1.399	1.207	1.135	1.037
0.50	3.563	1.910	1.603	1.470	1.240	1.156	1.042
0.60	4.176	2.070	1.682	1.541	1.273	1.175	1.047
0.70	4.870	2.237	1.798	1.614	1.306	1.196	1.052
0.80	5.663	2.415	1.901	1.688	1.338	1.215	1.057
0.90	6.570	2.604	2.006	1.763	1.371	1.235	1.062
1.00	7.605	2.805	2.117	1.842	1.404	1.254	1.067
1.10	8.795	3.019	2.233	1.924	1.437	1.274	1.071
1.20	10.155	3.250	2.355	2.008	1.471	1.294	1.076
1.30	11.718	3.497	2.483	2.096	1.506	1.314	1.080
1.40	13.513	3.761	2.617	2.187	1.540	1.334	1.085
1.50	15.569	4.045	2.758	2.282	1.576	1.354	1.089
1.60	17.928	4.351	2.907	2.380	1.613	1.374	1.094
1.70	20.639	4.680	3.064	2.484	1.650	1.395	1.098
1.80	23.749	5.034	3.229	2.592	1.688	1.416	1.103
1.90	27.318	5.414	3.403	2.704	1.728	1.438	1.107
2.00	31.398	5.825	3.588	2.822	1.767	1.459	1.112
2.10	36.079	6.268	3.783	2.945	1.808	1.481	1.116
2.20	41.444	6.745	3.989	3.074	1.850	1.504	1.121
2.30	47.586	7.260	4.208	3.209	1.893	1.526	1.125
2.40	54.611	7.815	4.438	3.351	1.937	1.549	1.130
2.50	62.661	8.415	4.683	3.498	1.982	1.572	1.134
2.60	71.861	9.061	4.941	3.670	2.029	1.596	1.139
2.70	82.366	9.759	5.214	3.816	2.076	1.620	1.144
2.80	94.377	10.512	5.504	3.986	2.125	1.645	1.148
2.90	108.115	11.3[illegible]	5.811	4.164	2.175	1.670	1.153
3.00	123.750	12.2[illegible]	6.137	4.351	2.226	1.695	1.158

FACTOR $\Upsilon_\eta(\nu)$ FOR ESTIMATION OF MEAN OF LOGNORMAL POPULATION

ν \ η	2	3	4	5	6	7	8	9	10	11	12
0.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.02	1.010	1.010	1.010	1.010	1.010	1.010	1.010	1.010	1.010	1.010	1.010
0.04	1.020	1.020	1.020	1.020	1.020	1.020	1.020	1.020	1.020	1.020	1.020
0.06	1.030	1.030	1.030	1.030	1.030	1.030	1.030	1.030	1.030	1.030	1.030
0.08	1.040	1.040	1.040	1.040	1.040	1.041	1.041	1.041	1.041	1.041	1.041
0.10	1.050	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051
0.12	1.061	1.061	1.061	1.061	1.061	1.061	1.061	1.061	1.061	1.062	1.062
0.14	1.071	1.071	1.071	1.072	1.072	1.072	1.072	1.072	1.072	1.072	1.072
0.16	1.081	1.082	1.082	1.082	1.082	1.082	1.082	1.083	1.083	1.083	1.083
0.18	1.091	1.092	1.092	1.093	1.093	1.093	1.093	1.093	1.093	1.093	1.094
0.20	1.102	1.102	1.103	1.103	1.104	1.104	1.104	1.104	1.104	1.104	1.104
0.3	1.154	1.156	1.157	1.158	1.158	1.159	1.159	1.159	1.160	1.160	1.160
0.4	1.207	1.210	1.212	1.214	1.215	1.216	1.216	1.217	1.217	1.217	1.218
0.5	1.260	1.266	1.269	1.272	1.273	1.275	1.276	1.276	1.277	1.278	1.278
0.6	1.315	1.323	1.328	1.332	1.334	1.336	1.337	1.338	1.339	1.340	1.341
0.7	1.371	1.382	1.389	1.393	1.397	1.399	1.401	1.403	1.404	1.406	1.406
0.8	1.427	1.442	1.451	1.457	1.462	1.465	1.468	1.470	1.472	1.473	1.475
0.9	1.485	1.503	1.515	1.523	1.529	1.533	1.537	1.540	1.542	1.544	1.546
1.0	1.543	1.565	1.580	1.591	1.598	1.604	1.608	1.612	1.615	1.618	1.620
1.1	1.602	1.630	1.648	1.661	1.670	1.677	1.682	1.687	1.691	1.694	1.697
1.2	1.662	1.696	1.718	1.733	1.744	1.752	1.759	1.765	1.770	1.774	1.777
1.3	1.724	1.764	1.789	1.807	1.820	1.831	1.839	1.846	1.851	1.856	1.860
1.4	1.786	1.832	1.862	1.884	1.900	1.912	1.922	1.930	1.936	1.942	1.947
1.5	1.848	1.903	1.938	1.963	1.981	1.996	2.007	2.017	2.025	2.032	2.037
1.6	1.912	1.975	2.015	2.044	2.066	2.082	2.095	2.107	2.116	2.124	2.131
1.7	1.977	2.049	2.095	2.128	2.153	2.172	2.188	2.201	2.212	2.221	2.229
1.8	2.043	2.124	2.177	2.214	2.243	2.265	2.283	2.298	2.310	2.321	2.330
1.9	2.110	2.201	2.260	2.303	2.336	2.361	2.382	2.399	2.413	2.425	2.436
2.0	2.178	2.280	2.347	2.395	2.431	2.460	2.484	2.503	2.519	2.533	2.545
2.1	2.247	2.360	2.435	2.489	2.530	2.563	2.589	2.611	2.630	2.645	2.659
2.2	2.317	2.442	2.526	2.586	2.632	2.669	2.699	2.723	2.744	2.762	2.778
2.3	2.388	2.526	2.618	2.686	2.737	2.778	2.811	2.839	2.863	2.883	2.900
2.4	2.460	2.612	2.714	2.788	2.846	2.891	2.928	2.959	2.986	3.008	3.028
2.5	2.533	2.699	2.812	2.894	2.957	3.008	3.049	3.084	3.113	3.138	3.160
2.6	2.607	2.789	2.912	3.003	3.073	3.128	3.174	3.213	3.245	3.274	3.298
2.7	2.682	2.880	3.015	3.114	3.191	3.253	3.304	3.346	3.382	3.414	3.441
2.8	2.759	2.973	3.120	3.229	3.314	3.382	3.437	3.484	3.524	3.559	3.589
2.9	2.836	3.068	3.228	3.347	3.440	3.514	3.576	3.627	3.671	3.710	3.743
3.0	2.914	3.166	3.339	3.469	3.570	3.651	3.718	3.775	3.824	3.866	3.902
3.1	2.994	3.265	3.453	3.593	3.703	3.792	3.866	3.928	3.981	4.028	4.068
3.2	3.075	3.366	3.569	3.721	3.841	3.938	4.018	4.086	4.145	4.195	4.240
3.3	3.157	3.469	3.688	3.853	3.983	4.088	4.176	4.250	4.314	4.369	4.418
3.4	3.240	3.574	3.810	3.988	4.129	4.243	4.338	4.419	4.489	4.549	4.603
3.5	3.324	3.682	3.935	4.127	4.279	4.403	4.506	4.594	4.670	4.736	4.794
3.6	3.409	3.792	4.063	4.270	4.434	4.568	4.680	4.775	4.858	4.929	4.993
3.7	3.495	3.903	4.194	4.416	4.593	4.738	4.859	4.962	5.052	5.130	5.198
3.8	3.583	4.017	4.329	4.567	4.757	4.913	5.044	5.156	5.252	5.337	5.412
3.9	3.672	4.134	4.466	4.721	4.925	5.093	5.234	5.355	5.460	5.552	5.633
4.0	3.762	4.252	4.607	4.880	5.099	5.279	5.431	5.562	5.675	5.774	5.862
4.1	3.853	4.373	4.751	5.042	5.277	5.471	5.634	5.775	5.897	6.004	6.099
4.2	3.946	4.496	4.898	5.209	5.460	5.668	5.844	5.995	6.127	6.242	6.345
4.3	4.040	4.622	5.049	5.380	5.649	5.872	6.060	6.223	6.364	6.489	6.599
4.4	4.135	4.750	5.203	5.556	5.843	6.081	6.283	6.458	6.610	6.744	6.863
4.5	4.231	4.881	5.361	5.736	6.042	6.297	6.513	6.700	6.863	7.008	7.136
4.6	4.328	5.014	5.522	5.921	6.247	6.519	6.750	6.950	7.125	7.281	7.419
4.7	4.427	5.149	5.687	6.111	6.457	6.747	6.995	7.209	7.397	7.563	7.711
4.8	4.527	5.288	5.856	6.305	6.674	6.983	7.247	7.476	7.677	7.855	8.014
4.9	4.629	5.428	6.029	6.505	6.896	7.225	7.507	7.751	7.966	8.157	8.328
5.0	4.732	5.572	6.205	6.709	7.124	7.474	7.774	8.036	8.265	8.470	8.652
5.1	4.836	5.718	6.385	6.919	7.359	7.731	8.050	8.329	8.574	8.792	8.988
5.2	4.941	5.868	6.570	7.134	7.600	7.996	8.335	8.631	8.893	9.126	9.335
5.3	5.048	6.018	6.759	7.354	7.847	8.266	8.628	8.944	9.222	9.471	9.695
5.4	5.156	6.172	6.951	7.579	8.102	8.546	8.930	9.265	9.563	9.828	10.07
5.5	5.266	6.329	7.149	7.811	8.363	8.833	9.240	9.593	9.914	10.20	10.45
5.6	5.376	6.489	7.350	8.048	8.631	9.129	9.561	9.940	10.28	10.58	10.85
5.7	5.489	6.652	7.555	8.290	8.905	9.433	9.890	10.29	10.65	10.97	11.26
5.8	5.603	6.818	7.766	8.539	9.188	9.745	10.23	10.66	11.04	11.38	11.68
5.9	5.718	6.987	7.980	8.794	9.478	10.07	10.58	11.03	11.44	11.80	
6.0	5.834	7.159	8.200	9.054	9.776	10.84	10.84	11.42	11.85		

SOME USEFUL FORMULAE

$$s^2 = \frac{1}{n-1}\sum (x-\bar{x})^2$$

$$s^2 = \frac{1}{n-1}\sum x^2 - n\bar{x}^2$$

$$T = \frac{\bar{x}-\mu}{s\sqrt{n}}$$

$$P = PV\frac{i}{1-(1+i)^n}$$

$$PV = P(1-(1+i)^n)/i$$

$$b = \frac{n\sum xy - \sum x\sum y}{n\sum x^2 - (\sum x)^2}$$

$$r = \frac{\sum xy - \sum x\sum y/n}{[\sum x^2 - (\sum x)^2/n][\sum y^2 - (\sum y)^2/n]}$$

$$\gamma(h) = C(\frac{3h}{2a} - \frac{h^3}{2a^3})$$

$$\gamma(h) = C(1-\exp(\frac{-h}{a}))$$

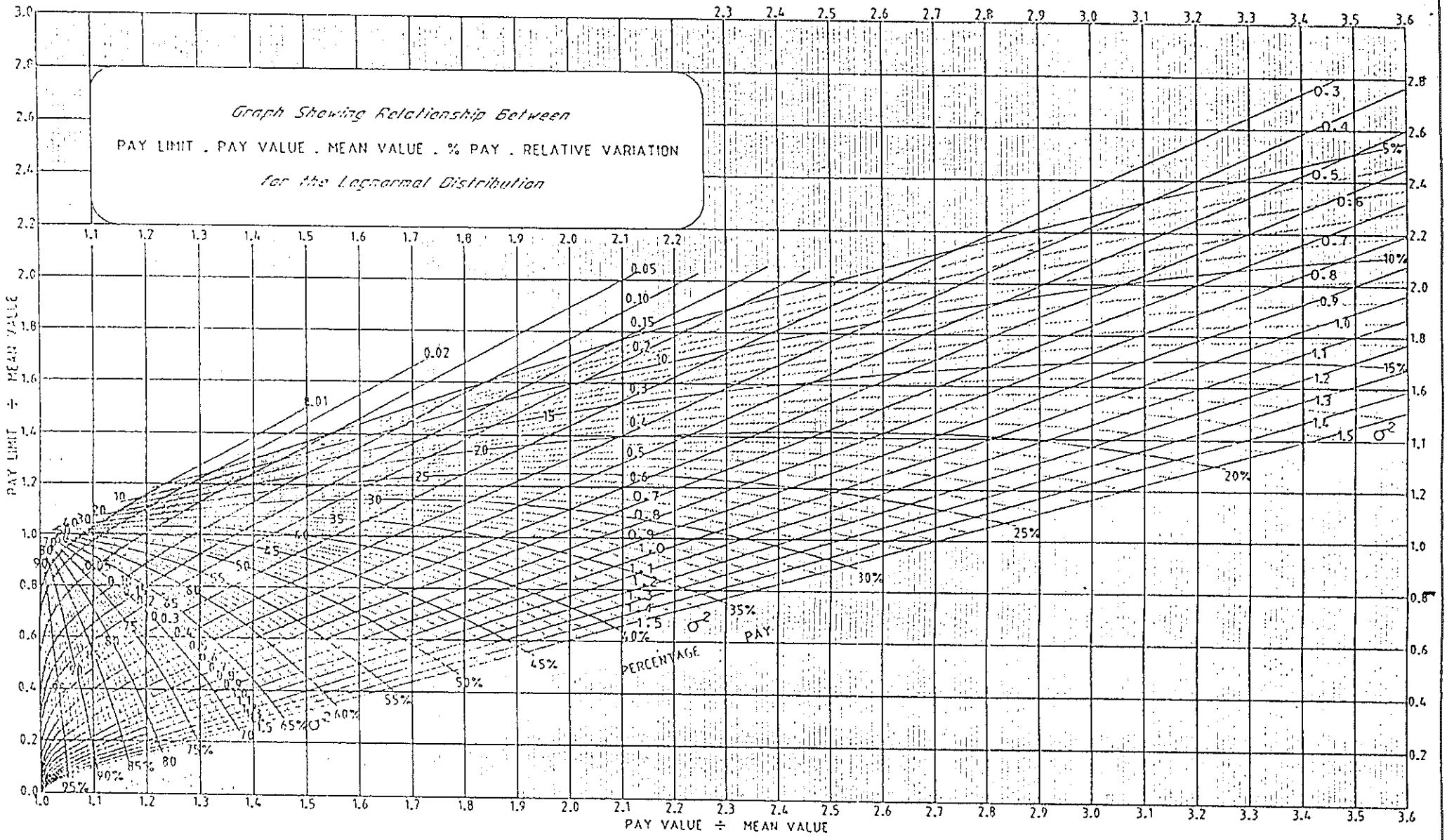
$$T = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

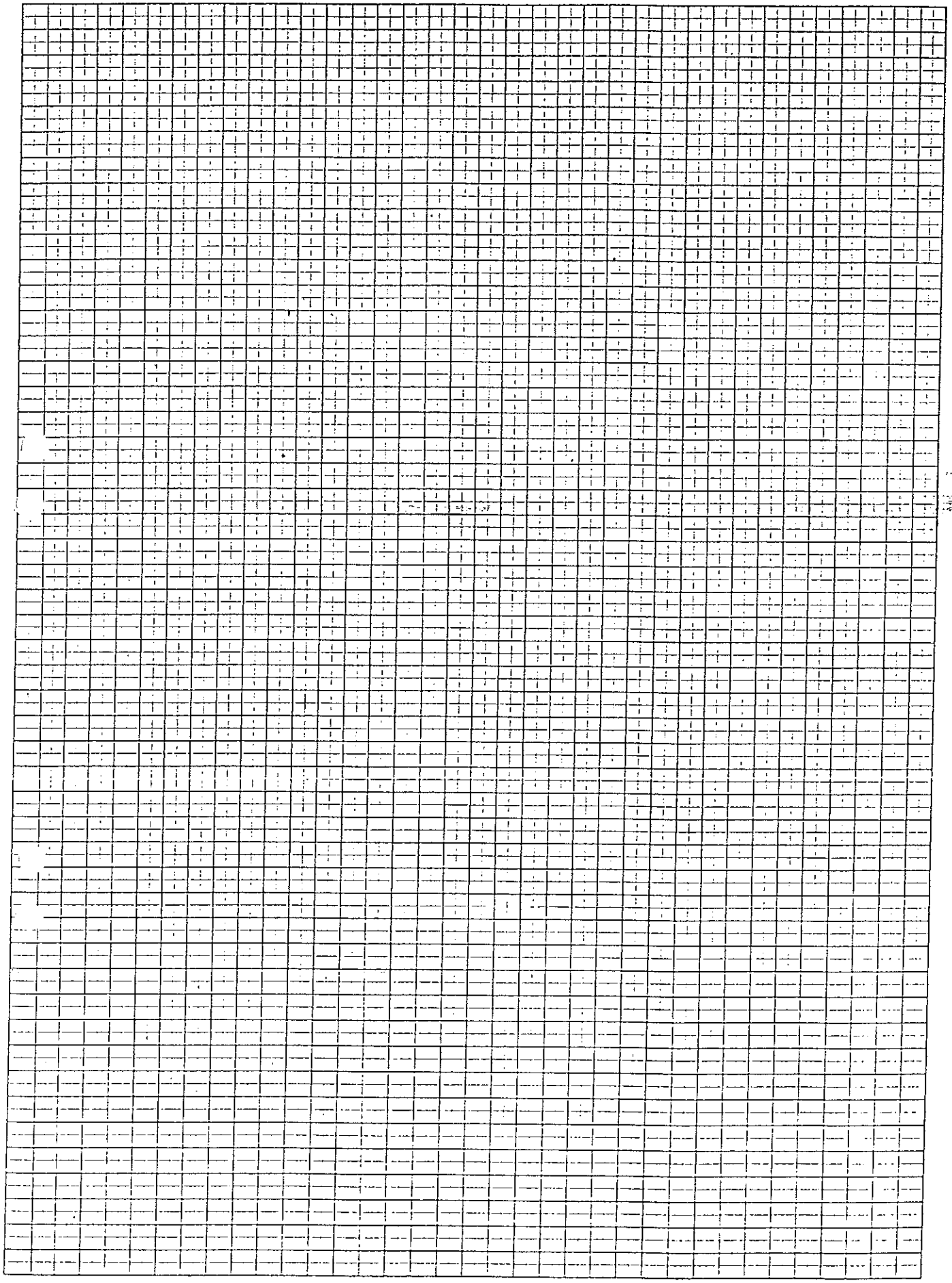
$$PV = A/(1+r)^n$$

$$\theta = \frac{Mx^2 - x_p x_{1-p}}{x_p + x_{1-p} - 2Mx}$$

$$2\gamma(h) = \frac{1}{n}\sum [g(x) - g(x+h)]$$

$$\sigma^c = \hat{s}_y\sqrt{1-r^2}$$


Graph Showing Relationship Between
PAY LIMIT . PAY VALUE . MEAN VALUE . % PAY . RELATIVE VARIATION
for the Lognormal Distribution
PAY LIMIT ÷ MEAN VALUE
PAY VALUE ÷ MEAN VALUE
PERCENTAGE PAY
σ²
GR.L.20
4TH NOV, 1953



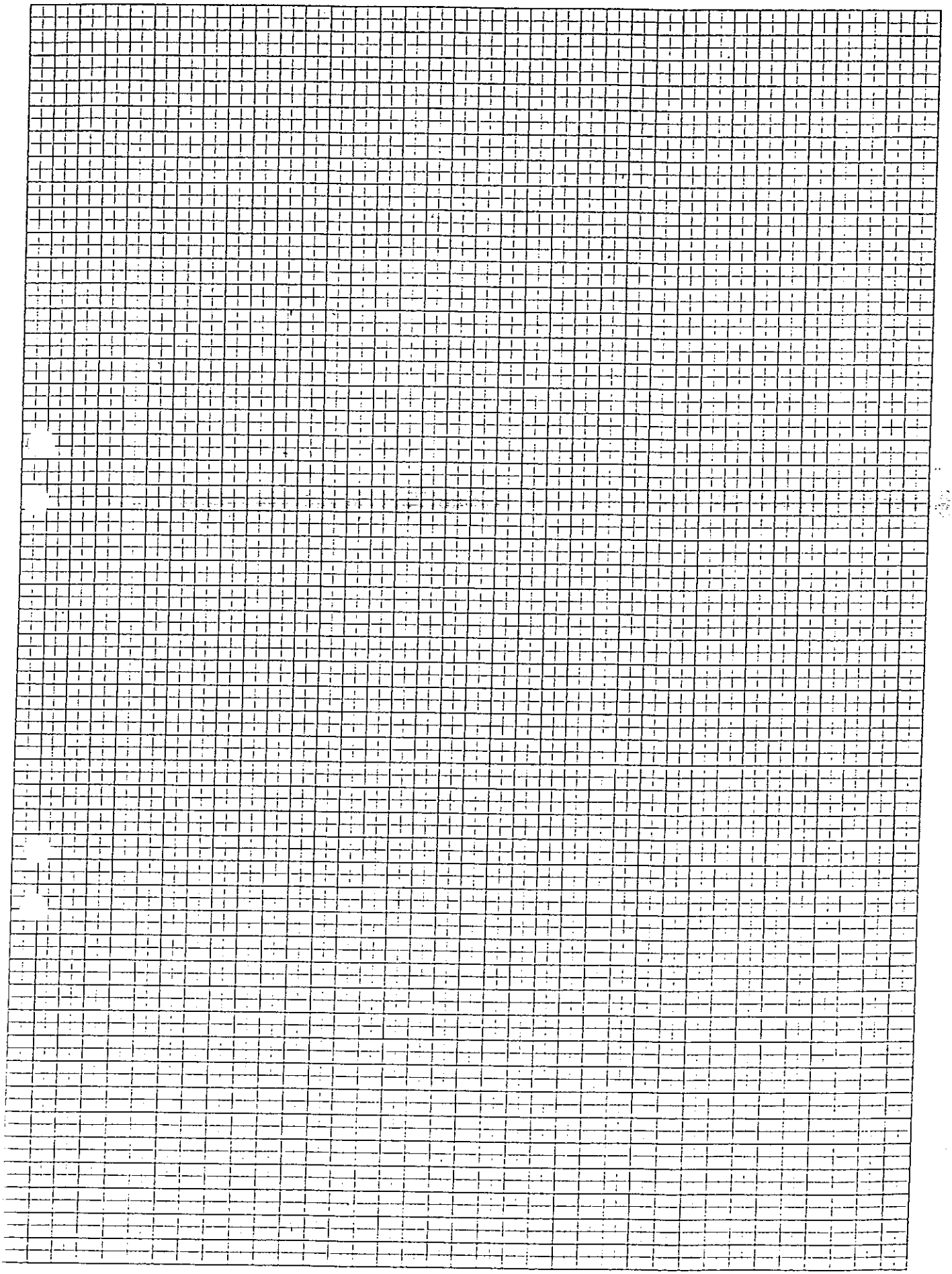
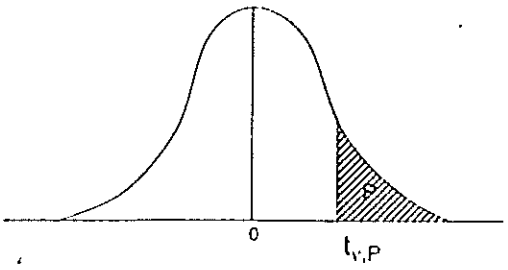


TABLE 3

The t-Distribution :
Upper Probability Points
$P = P(t \geq t_{\nu,P}) = P(t \leq -t_{\nu,P})$
with $t_{\nu,P} = -t_{\nu,1-P}$ so that
$P(|t| \geq t_{\nu,P}) = 2P, \quad t_{\nu,P} > 0.$

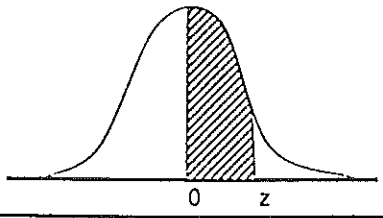
Entries in the table are the values $t_{\nu,P}$ of the t-distribution for various degrees of freedom ν and one tailed probabilities P.



ν \ P	0.25	0.10	0.05	0.025	0.01	0.005
1	1.000	3.078	6.314	12.706	31.821	63.657
2	0.816	1.886	2.920	4.303	6.965	9.925
3	0.765	1.638	2.353	3.182	4.541	5.841
4	0.741	1.533	2.132	2.776	3.747	4.604
5	0.727	1.476	2.015	2.571	3.365	4.032
6	0.718	1.440	1.943	2.447	3.143	3.707
7	0.711	1.415	1.895	2.365	2.998	3.499
8	0.706	1.397	1.860	2.306	2.896	3.355
9	0.703	1.383	1.833	2.262	2.821	3.250
10	0.700	1.372	1.812	2.228	2.764	3.169
11	0.697	1.363	1.796	2.201	2.718	3.106
12	0.695	1.356	1.782	2.179	2.681	3.055
13	0.694	1.350	1.771	2.160	2.650	3.012
14	0.692	1.345	1.761	2.145	2.624	2.977
15	0.691	1.341	1.753	2.131	2.602	2.947
16	0.690	1.337	1.746	2.120	2.583	2.921
17	0.689	1.333	1.740	2.110	2.567	2.898
18	0.688	1.330	1.734	2.101	2.552	2.878
19	0.688	1.328	1.729	2.093	2.539	2.861
20	0.687	1.325	1.725	2.086	2.528	2.845
21	0.686	1.323	1.721	2.080	2.518	2.831
22	0.686	1.321	1.717	2.074	2.508	2.819
23	0.685	1.319	1.714	2.069	2.500	2.807
24	0.685	1.318	1.711	2.064	2.492	2.797
25	0.684	1.316	1.708	2.060	2.485	2.787
26	0.684	1.315	1.706	2.056	2.479	2.779
27	0.684	1.314	1.703	2.052	2.473	2.771
28	0.683	1.313	1.701	2.048	2.467	2.763
29	0.683	1.311	1.699	2.045	2.462	2.756
30	0.683	1.310	1.697	2.042	2.457	2.750
35	0.682	1.306	1.690	2.030	2.438	2.724
40	0.681	1.303	1.684	2.021	2.423	2.704
60	0.679	1.296	1.671	2.000	2.390	2.660
100	0.677	1.290	1.660	1.984	2.364	2.626
∞	0.675	1.282	1.645	1.960	2.326	2.576

AREAS under the STANDARD NORMAL CURVE from 0 to z

TABLE 1



z	0	1	2	3	4	5	6	7	8	9
0 0	0.0000	0 0040	0.0080	0 0120	0.0160	0.0199	0.0239	0 0279	0.0319	0 0359
0 1	0.0398	0.0438	0.0478	0.0517	0 0557	0.0596	0.0636	0 0675	0.0714	0 0754
0 2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0 1141
0 3	0.1179	0 1217	0.1255	0.1293	0.1331	0.1368	0.1406	0 1443	0.1480	0 1517
0 4	0 1554	0.1591	0.1628	0.1664	0.1700	0 1736	0.1772	0 1803	0.1844	0 1879
0 5	0 1915	0 1950	0.1985	0.2019	0.2054	0.2088	0.2123	0 2157	0 2190	0 2224
0 6	0 2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0 2486	0.2518	0 2549
0 7	0 2580	0 2612	0.2642	0 2673	0 2704	0 2734	0 2764	0 2794	0 2823	0 2852
0 8	0 2881	0 2910	0 2939	0 2967	0 2996	0.3023	0.3051	0 3078	0 3106	0 3133
0 9	0 3159	0 3186	0 3212	0.3238	0.3264	0.3289	0.3315	0 3340	0 3365	0 3389
1 0	0 3413	0 3438	0 3461	0.3485	0 3508	0 3531	0 3554	0 3577	0 3599	0 3621
1 1	0 3643	0 3665	0 3686	0.3708	0 3729	0.3749	0.3770	0 3790	0 3810	0 3830
1 2	0 3849	0 3869	0 3888	0.3907	0 3925	0 3944	0.3962	0 3980	0 3997	0 4015
1 3	0 4032	0 4049	0 4066	0 4082	0 4099	0 4115	0 4131	0 4147	0 4162	0 4177
1 4	0 4192	0 4207	0 4222	0 4236	0 4251	0 4265	0.4279	0 4292	0 4306	0 4319
1 5	0 4332	0 4345	0.4357	0 4370	0 4382	0 4394	0 4406	0 4418	0.4429	0 4441
1 6	0 4452	0 4463	0 4474	0 4484	0 4495	0 4505	0 4515	0 4525	0 4535	0 4545
1 7	0 4554	0 4564	0 4573	0 4582	0 4591	0 4599	0.4608	0 4616	0 4625	0 4633
1 8	0 4641	0 4649	0 4656	0.4664	0 4671	0 4678	0.4686	0 4693	0 4699	0 4706
1 9	0 4713	0 4719	0 4726	0 4732	0 4738	0 4744	0 4750	0 4756	0 4761	0 4767
2 0	0 4772	0 4778	0 4783	0 4788	0 4793	0 4798	0 4803	0 4808	0 4812	0 4817
2 1	0 4821	0 4826	0 4830	0 4834	0 4838	0 4842	0 4846	0 4350	0.4854	0 4857
2 2	0 4861	0 4864	0.4868	0 4871	0 4375	0 4878	0.4881	0 4884	0 4887	0 4890
2 3	0 4893	0 4896	0 4898	0 4901	0 4904	0 4906	0 4909	0 4911	0 4913	0 4916
2 4	0 4918	0 4920	0 4922	0 4925	0 4927	0 4929	0 4931	0 4932	0 4934	0 4936
2 5	0 4938	0 4940	0 4941	0 4943	0 4945	0 4946	0.4948	0 4949	0 4951	0 4952
2 6	0 4953	0 4955	0 4956	0 4957	0 4959	0 4960	0.4961	0 4962	0 4963	0 4964
2 7	0 4965	0 4966	0 4967	0.4968	0 4969	0 4970	0 4971	0 4972	0 4973	0 4974
2 8	0 4974	0 4975	0 4976	0 4977	0 4977	0 4978	0 4979	0 4979	0 4980	0 4981
2 9	0 4981	0 4982	0 4982	0 4983	0 4984	0 4984	0.4985	0 4985	0 4986	0 4986
3 0	0 4987	0 4987	0 4987	0 4988	0 4988	0 4989	0 4989	0 4989	0 4990	0 4990
3 1	0 4990	0 4991	0 4991	0 4991	0 4992	0 4992	0 4992	0 4992	0 4993	0 4993
3 2	0 4993	0 4993	0 4994	0 4994	0 4994	0 4994	0 4994	0 4995	0 4995	0 4995
3 3	0 4995	0 4995	0 4995	0 4996	0 4996	0 4996	0 4996	0 4996	0 4996	0 4997
3 4	0 4997	0 4997	0 4997	0 4997	0 4997	0 4997	0 4997	0 4997	0 4997	0 4998
3 5	0 4998	0 4998	0 4998	0 4998	0 4998	0 4998	0 4998	0 4998	0 4998	0 4998
3 6	0 4998	0 4998	0 4999	0 4999	0 4999	0 4999	0 4999	0 4999	0 4999	0 4999
3 7	0 4999	0 4999	0 4999	0 4999	0 4999	0 4999	0 4999	0 4999	0 4999	0 4999
3 8	0 4999	0 4999	0 4999	0 4999	0 4999	0 4999	0 4999	0 4999	0 4999	0 4999
3 9	0 5000	0 5000	0 5000	0 5000	0 5000	0 5000	0 5000	0 5000	0 5000	0 5000

TABLE FOR THE COMPUTATION OF TONNAGE AND GRADE CUT-OFF FOR THE NORMAL DISTRIBUTION				
Cut-off below the mean		Reduced cut-off = Z PV=MV + $\omega\sigma$	Cut-off above the mean	
Tonnage proportion	ω – factor		ω – factor	Tonnage proportion
50.00	0.798	0.00	0.798	50.00
51.99	0.766	0.05	0.830	48.01
53.98	0.735	0.10	0.862	46.02
55.96	0.705	0.15	0.896	44.04
57.93	0.675	0.20	0.929	42.07
59.87	0.646	0.25	0.963	40.13
61.79	0.617	0.30	0.998	38.21
63.68	0.589	0.35	1.033	36.32
65.54	0.562	0.40	1.069	34.46
67.36	0.535	0.45	1.104	32.64
69.15	0.509	0.50	1.141	30.85
70.88	0.484	0.55	1.178	29.12
72.57	0.459	0.60	1.215	27.43
74.22	0.435	0.65	1.252	25.78
75.80	0.412	0.70	1.290	24.20
77.34	0.389	0.75	1.329	22.66
78.81	0.367	0.80	1.367	21.19
80.23	0.346	0.85	1.406	19.77
81.59	0.326	0.90	1.445	18.41
82.89	0.306	0.95	1.485	17.11
84.13	0.288	1.00	1.525	15.87
85.31	0.269	1.05	1.565	14.69
86.43	0.252	1.10	1.605	13.57
87.49	0.235	1.15	1.646	12.51
88.49	0.219	1.20	1.687	11.51
89.44	0.204	1.25	1.728	10.56
90.32	0.190	1.30	1.770	9.68
91.15	0.176	1.35	1.812	8.85
91.92	0.163	1.40	1.854	8.08
92.65	0.150	1.45	1.896	7.35
93.32	0.139	1.50	1.938	6.68
93.94	0.128	1.55	1.981	6.06
94.52	0.117	1.60	2.024	5.48
95.05	0.108	1.65	2.067	4.95
95.54	0.098	1.70	2.110	4.46
95.99	0.090	1.75	2.153	4.01
96.41	0.082	1.80	2.197	3.59
96.78	0.074	1.85	2.241	3.22
97.13	0.068	1.90	2.284	2.87
97.44	0.061	1.95	2.329	2.56
97.72	0.055	2.00	2.373	2.28
97.98	0.050	2.05	2.417	2.02
98.21	0.045	2.10	2.462	1.79
98.42	0.040	2.15	2.506	1.58
98.61	0.036	2.20	2.551	1.39
98.78	0.032	2.25	2.596	1.22