REPUBLIC OF SOUTH AFRICA
DEPARTMENT OF MINERALS AND ENERGY
EXAMINATION FOR THE MINE SURVEYOR'S CERTIFICATE OF COMPETENCY

DATE: 19 April 2007 (Thursday)	TOTAL MARKS: 100
TIME: 12:30 – 15:30 (3 Hours)	TO PASS: 50

MINING ECONOMICS II

NOTE:

1. Any pocket calculator may be used and intermediate results need not be shown. The make and model number of the calculator used must be noted on the front cover of the answer book.
2. Tables that may be used are attached hereto.
3. Graph, Probability and Log paper will be supplied if required
4. Your examination number must be written on all graph paper and loose sheets that are handed in with your examination script.

Question 1

Explain what is meant by the following terms:

a) Cross Validation (2)
b) Semi-variogram (2)
c) Simple Interest (2)
d) Compound Interest (2)
e) Regularisation (2)

[10]

Question 2

The following widths, in centimetres, were taken from an ore body, known to be normally distributed:

177	188	190	182	177	174	184	182	177	180	183	177	185
193	178	191	175	172	172	182	170	181	174	184	182	

Determine the following:

a) Mean, mode, median and range
b) Variance and standard deviation of the samples
c) Coefficient of variation of the samples
d) Standard deviation of the population
e) 90% confidence limits for the population standard deviation
f) 90% confidence limits for the population mean

[25]

Question 3

The following details, based on 200 two ton samples, are available:

Mean Value	31,0% silver
Standard deviation	6,5% silver
Size of deposit	20 million tons of ore
Minimum quantity that can be transported	30 tons

a) Determine grade tonnage curves for cut-off values ranging from 28% to 34% silver
b) Read from the graphs, the grade and payable tons for the cut-off values of 30,5% and 32,5% silver

[20]

Question 4

The table listed below shows the X values of a mineral together with its corresponding Y values

X	3	9	11	12	14	16	21	21
Y	11	14	15	16	19	15	20	20

Determine the following:

a) The regression line to estimate Y values from the X values
b) The correlation coefficient between the two variables
c) Standard deviations for the X, Y and Z and error distributions
d) The effective pay limit if the official pay limit is 16,0
e) The Z value for an X value of 11,5
f) The probability of the Z value above proving to be unpay
g) The 90% confidence limits for the regressed value

[15]

Question 5

The following borehole values were obtained from a mineral deposit where the additive constant is known to be 1 000 cm.g/t.

(Values in cm.g/t)

330 360 400 600 820 970 1 350 1 840 2 470 4 050 8 700

Calculate:

a) The mean value for the deposit
b) The 95% confidence limits for the mean value

[20]

Question 6

A mine section carries a fixed cost of R2.5 million with a variable cost of R70 per gram produced. If the expected revenue is R100 per gram,

a) What is the break even tonnage if the grade is 6 g/t?
b) What does this break even tonnage become if the actual revenue received is 10% less than expected?

[10]

Total Marks [100]

FACTOR $\Upsilon_\eta(\nu)$ FOR ESTIMATION OF MEAN OF LOGNORMAL POPULATION

ν \ η	2	3	4	5	6	7	8	9	10	11	12
0.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.02	1.010	1.010	1.010	1.010	1.010	1.010	1.010	1.010	1.010	1.010	1.010
0.04	1.020	1.020	1.020	1.020	1.020	1.020	1.020	1.020	1.020	1.020	1.020
0.06	1.030	1.030	1.030	1.030	1.030	1.030	1.030	1.030	1.030	1.030	1.030
0.08	1.040	1.040	1.040	1.040	1.040	1.041	1.041	1.041	1.041	1.041	1.041
0.10	1.050	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051
0.12	1.061	1.061	1.061	1.061	1.061	1.061	1.061	1.061	1.061	1.062	1.062
0.14	1.071	1.071	1.071	1.072	1.072	1.072	1.072	1.072	1.072	1.072	1.072
0.16	1.081	1.082	1.082	1.082	1.082	1.082	1.082	1.083	1.083	1.083	1.083
0.18	1.091	1.092	1.092	1.093	1.093	1.093	1.093	1.093	1.093	1.093	1.094
0.20	1.102	1.102	1.103	1.103	1.104	1.104	1.104	1.104	1.104	1.104	1.104
0.3	1.154	1.156	1.157	1.158	1.158	1.159	1.159	1.159	1.160	1.160	1.160
0.4	1.207	1.210	1.212	1.214	1.215	1.216	1.216	1.217	1.217	1.217	1.218
0.5	1.260	1.266	1.269	1.272	1.273	1.275	1.276	1.276	1.277	1.278	1.278
0.6	1.315	1.323	1.328	1.332	1.334	1.336	1.337	1.338	1.339	1.340	1.341
0.7	1.371	1.382	1.389	1.393	1.397	1.399	1.401	1.403	1.404	1.406	1.406
0.8	1.427	1.442	1.451	1.457	1.462	1.465	1.468	1.470	1.472	1.473	1.475
0.9	1.485	1.503	1.515	1.523	1.529	1.533	1.537	1.540	1.542	1.544	1.546
1.0	1.543	1.566	1.580	1.591	1.598	1.604	1.608	1.612	1.615	1.618	1.620
1.1	1.602	1.630	1.648	1.661	1.670	1.677	1.682	1.687	1.691	1.694	1.697
1.2	1.662	1.696	1.718	1.733	1.744	1.752	1.759	1.765	1.770	1.774	1.777
1.3	1.724	1.764	1.789	1.807	1.820	1.831	1.839	1.846	1.851	1.856	1.860
1.4	1.786	1.832	1.862	1.884	1.900	1.912	1.922	1.930	1.936	1.942	1.947
1.5	1.848	1.903	1.938	1.963	1.981	1.996	2.007	2.017	2.025	2.032	2.037
1.6	1.912	1.975	2.015	2.044	2.066	2.082	2.096	2.107	2.116	2.124	2.131
1.7	1.977	2.049	2.095	2.128	2.153	2.172	2.188	2.201	2.212	2.221	2.229
1.8	2.043	2.124	2.177	2.214	2.243	2.265	2.283	2.298	2.310	2.321	2.330
1.9	2.110	2.201	2.260	2.303	2.336	2.361	2.382	2.399	2.413	2.425	2.436
2.0	2.178	2.280	2.347	2.395	2.431	2.460	2.484	2.503	2.519	2.533	2.545
2.1	2.247	2.360	2.435	2.489	2.530	2.563	2.589	2.611	2.630	2.645	2.659
2.2	2.317	2.442	2.526	2.586	2.632	2.669	2.698	2.723	2.744	2.762	2.778
2.3	2.388	2.526	2.618	2.686	2.737	2.778	2.811	2.839	2.863	2.883	2.900
2.4	2.460	2.612	2.714	2.788	2.846	2.891	2.928	2.959	2.986	3.008	3.028
2.5	2.533	2.699	2.812	2.894	2.957	3.008	3.049	3.084	3.113	3.138	3.160
2.6	2.607	2.789	2.912	3.003	3.073	3.128	3.174	3.213	3.245	3.274	3.298
2.7	2.682	2.880	3.015	3.114	3.191	3.253	3.304	3.346	3.382	3.414	3.441
2.8	2.759	2.973	3.120	3.229	3.314	3.382	3.437	3.484	3.524	3.559	3.589
2.9	2.836	3.068	3.228	3.347	3.440	3.514	3.576	3.627	3.671	3.710	3.743
3.0	2.914	3.166	3.339	3.469	3.570	3.651	3.718	3.775	3.824	3.866	3.902
3.1	2.994	3.265	3.453	3.593	3.703	3.792	3.866	3.928	3.981	4.028	4.068
3.2	3.075	3.366	3.569	3.721	3.841	3.938	4.018	4.086	4.145	4.195	4.240
3.3	3.157	3.469	3.688	3.853	3.983	4.088	4.176	4.250	4.314	4.369	4.418
3.4	3.240	3.574	3.810	3.988	4.129	4.243	4.338	4.419	4.489	4.549	4.603
3.5	3.324	3.682	3.935	4.127	4.279	4.403	4.506	4.594	4.670	4.736	4.794
3.6	3.409	3.792	4.063	4.270	4.434	4.568	4.680	4.775	4.858	4.929	4.993
3.7	3.496	3.903	4.194	4.416	4.593	4.738	4.859	4.962	5.052	5.130	5.198
3.8	3.583	4.017	4.329	4.567	4.757	4.913	5.044	5.156	5.252	5.337	5.412
3.9	3.672	4.134	4.466	4.721	4.925	5.093	5.234	5.355	5.460	5.552	5.633
4.0	3.762	4.252	4.607	4.880	5.099	5.279	5.431	5.562	5.675	5.774	5.862
4.1	3.853	4.373	4.751	5.042	5.277	5.471	5.634	5.775	5.897	6.004	6.099
4.2	3.946	4.496	4.898	5.209	5.460	5.668	5.844	5.995	6.127	6.242	6.345
4.3	4.040	4.622	5.049	5.380	5.649	5.872	6.060	6.223	6.364	6.489	6.599
4.4	4.135	4.750	5.203	5.556	5.843	6.081	6.283	6.458	6.610	6.744	6.863
4.5	4.231	4.881	5.361	5.736	6.042	6.297	6.513	6.700	6.863	7.008	7.136
4.6	4.328	5.014	5.522	5.921	6.247	6.519	6.750	6.950	7.126	7.281	7.419
4.7	4.427	5.149	5.687	6.111	6.457	6.747	6.995	7.209	7.397	7.563	7.711
4.8	4.527	5.288	5.856	6.305	6.674	6.983	7.247	7.476	7.677	7.855	8.014
4.9	4.629	5.428	6.029	6.505	6.896	7.225	7.507	7.751	7.966	8.157	8.328
5.0	4.732	5.572	6.205	6.709	7.124	7.474	7.774	8.036	8.265	8.470	8.652
5.1	4.836	5.718	6.386	6.919	7.359	7.731	8.050	8.329	8.574	8.792	8.988
5.2	4.941	5.866	6.570	7.134	7.600	7.995	8.335	8.631	8.893	9.126	9.335
5.3	5.048	6.018	6.759	7.354	7.847	8.266	8.628	8.944	9.222	9.471	9.695
5.4	5.156	6.172	6.951	7.579	8.102	8.546	8.930	9.265	9.563	9.828	10.07
5.5	5.266	6.329	7.148	7.811	8.363	8.833	9.240	9.598	9.914	10.20	10.45
5.6	5.376	6.489	7.350	8.048	8.631	9.129	9.561	9.940	10.28	10.58	10.85
5.7	5.489	6.652	7.555	8.290	8.906	9.433	9.890	10.29	10.65	10.97	11.26
5.8	5.603	6.818	7.766	8.539	9.188	9.745	10.23	10.66	11.04	11.38	11.68
5.9	5.718	6.987	7.980	8.794	9.478	10.07	10.58	11.03	11.44	11.80	
6.0	5.834	7.159	8.200	9.054	9.776	10.84	10.84	11.42	11.85		

LOWER 95%

Factor $b_{0.05}(v;n)$ for estimation of one sided lower 95% confidence limits of the mean of a lognormal population

v \ n	5	10	15	20	50	100	1000
0.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.02	0.8978	0.9333	0.9458	0.9540	0.9697	0.9782	0.9927
0.04	0.8589	0.9071	0.9246	0.9344	0.9573	0.9692	0.9895
0.06	0.8302	0.8874	0.9079	0.9200	0.9478	0.9622	0.9872
0.08	0.8070	0.8708	0.8943	0.9077	0.9398	0.9564	0.9852
0.10	0.7870	0.8563	0.8821	0.8972	0.9328	0.9512	0.9833
0.12	0.7693	0.8439	0.8716	0.8878	0.9264	0.9464	0.9817
0.14	0.7535	0.8323	0.8617	0.8790	0.9204	0.9420	0.9801
0.16	0.7389	0.8216	0.8527	0.8709	0.9149	0.9380	0.9787
0.18	0.7255	0.8116	0.8442	0.8632	0.9097	0.9341	0.9773
0.20	0.7129	0.8023	0.8360	0.8558	0.9048	0.9304	0.9760
0.30	0.6605	0.7618	0.8008	0.8243	0.8828	0.9139	0.9701
0.40	0.6187	0.7284	0.7717	0.7981	0.8639	0.8996	0.9648
0.50	0.5838	0.6995	0.7462	0.7744	0.8470	0.8867	0.9600
0.60	0.5538	0.6739	0.7270	0.7534	0.8313	0.8741	0.9554
0.70	0.5277	0.6508	0.7020	0.7338	0.8168	0.8632	0.9511
0.80	0.5044	0.6297	0.6825	0.7156	0.8030	0.8525	0.9470
0.90	0.4836	0.6103	0.6646	0.6987	0.7899	0.8421	0.9429
1.00	0.4650	0.5923	0.6476	0.6826	0.7774	0.8322	0.9389
1.10	0.4481	0.5756	0.6317	0.6674	0.7654	0.8226	0.9351
1.20	0.4328	0.5599	0.6165	0.6530	0.7538	0.8133	0.9313
1.30	0.4189	0.5452	0.6023	0.6393	0.7426	0.8042	0.9276
1.40	0.4062	0.5315	0.5888	0.6262	0.7318	0.7954	0.9240
1.50	0.3946	0.5186	0.5760	0.6137	0.7214	0.7868	0.9203
1.60	0.3840	0.5065	0.5637	0.6018	0.7112	0.7784	0.9168
1.70	0.3743	0.4950	0.5521	0.5904	0.7014	0.7702	0.9133
1.80	0.3655	0.4842	0.5410	0.5794	0.6918	0.7622	0.9098
1.90	0.3574	0.4740	0.5305	0.5688	0.6825	0.7544	0.9064
2.00	0.3501	0.4644	0.5203	0.5587	0.6734	0.7466	0.9030
2.10	0.3433	0.4552	0.5106	0.5489	0.6646	0.7391	0.8996
2.20	0.3372	0.4466	0.5014	0.5395	0.6560	0.7317	0.8962
2.30	0.3316	0.4385	0.4925	0.5304	0.6476	0.7245	0.8929
2.40	0.3266	0.4308	0.4840	0.5217	0.6394	0.7173	0.8896
2.50	0.3220	0.4234	0.4759	0.5133	0.6314	0.7104	0.8864
2.60	0.3179	0.4166	0.4681	0.5044	0.6236	0.7035	0.8831
2.70	0.3142	0.4100	0.4606	0.4974	0.6160	0.6967	0.8799
2.80	0.3110	0.4039	0.4535	0.4899	0.6085	0.6901	0.8767
2.90	0.3081	0.3981	0.4467	0.4826	0.6012	0.6836	0.8736
3.00	0.3055	0.3926	0.4401	0.4756	0.5941	0.6772	0.8704

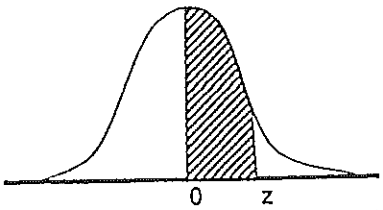
UPPER 95%

Factor $b_{0.95}(v;n)$ for estimation of one sided upper 95% confidence limits of the mean of a lognormal population

v \ n	5	10	15	20	50	100	1000
0.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.02	1.241	1.117	1.084	1.067	1.038	1.026	1.007
0.04	1.362	1.171	1.122	1.099	1.055	1.037	1.011
0.06	1.466	1.216	1.154	1.124	1.069	1.046	1.013
0.08	1.561	1.256	1.181	1.146	1.080	1.053	1.015
0.10	1.652	1.293	1.207	1.166	1.091	1.060	1.017
0.12	1.740	1.327	1.230	1.184	1.100	1.066	1.019
0.14	1.827	1.361	1.253	1.202	1.109	1.072	1.020
0.16	1.914	1.393	1.274	1.219	1.118	1.078	1.022
0.18	1.999	1.425	1.295	1.236	1.126	1.084	1.023
0.20	2.087	1.455	1.316	1.252	1.135	1.089	1.025
0.30	2.532	1.606	1.415	1.328	1.172	1.113	1.031
0.40	3.019	1.756	1.509	1.399	1.207	1.135	1.037
0.50	3.563	1.910	1.603	1.470	1.240	1.156	1.042
0.60	4.176	2.070	1.682	1.541	1.273	1.175	1.047
0.70	4.870	2.237	1.798	1.614	1.306	1.196	1.052
0.80	5.663	2.415	1.901	1.688	1.338	1.215	1.057
0.90	6.570	2.604	2.006	1.763	1.371	1.235	1.062
1.00	7.605	2.805	2.117	1.842	1.404	1.254	1.067
1.10	8.795	3.019	2.233	1.924	1.437	1.274	1.071
1.20	10.155	3.250	2.355	2.008	1.471	1.294	1.076
1.30	11.718	3.497	2.483	2.096	1.506	1.314	1.080
1.40	13.513	3.761	2.617	2.187	1.540	1.334	1.085
1.50	15.569	4.045	2.758	2.282	1.576	1.354	1.089
1.60	17.928	4.351	2.907	2.380	1.613	1.374	1.094
1.70	20.639	4.680	3.064	2.484	1.650	1.395	1.098
1.80	23.749	5.034	3.229	2.592	1.688	1.416	1.103
1.90	27.318	5.414	3.403	2.704	1.728	1.438	1.107
2.00	31.398	5.825	3.588	2.822	1.767	1.459	1.112
2.10	36.079	6.268	3.783	2.945	1.808	1.481	1.116
2.20	41.444	6.745	3.989	3.074	1.850	1.504	1.121
2.30	47.586	7.260	4.208	3.209	1.893	1.526	1.125
2.40	54.611	7.815	4.438	3.351	1.937	1.549	1.130
2.50	62.661	8.415	4.683	3.498	1.982	1.572	1.134
2.60	71.861	9.061	4.941	3.670	2.029	1.596	1.139
2.70	82.366	9.759	5.214	3.816	2.076	1.620	1.144
2.80	94.377	10.512	5.504	3.986	2.125	1.645	1.148
2.90	108.115	11.326	5.811	4.164	2.175	1.670	1.153
3.00	123.750	12.206	6.137	4.351	2.226	1.695	1.158

AREAS under the
STANDARD
NORMAL CURVE
from 0 to z

TABLE 1



Z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1803	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.5157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

TABLE 5

TABLE FOR THE COMPUTATION OF TONNAGE AND GRADE CUT-OFF FOR THE NORMAL DISTRIBUTION

Cut-off below the mean		Reduced cut-off = Z	Cut-off above the mean	
Tonnage proportion	ω - factor	PV=MV+ ωσ	ω - factor	Tonnage proportion
50.00	0.798	0.00	0.798	50.00
51.99	0.766	0.05	0.830	48.01
53.98	0.735	0.10	0.863	46.02
55.96	0.705	0.15	0.896	44.04
57.93	0.675	0.20	0.929	42.07
59.87	0.645	0.25	0.964	40.13
61.79	0.617	0.30	0.998	38.21
63.68	0.589	0.35	1.034	36.32
65.54	0.562	0.40	1.069	34.45
67.36	0.535	0.45	1.106	32.64
69.15	0.509	0.50	1.142	30.85
70.88	0.484	0.55	1.180	29.12
72.57	0.459	0.60	1.217	27.43
74.22	0.435	0.65	1.256	25.78
75.80	0.411	0.70	1.295	24.20
77.34	0.389	0.75	1.334	22.66
78.81	0.367	0.80	1.375	21.19
80.23	0.346	0.85	1.415	19.77
81.59	0.326	0.90	1.457	18.41
82.89	0.306	0.95	1.499	17.11
84.13	0.287	1.00	1.542	15.87
85.31	0.269	1.05	1.586	14.69
86.43	0.251	1.10	1.631	13.57
87.49	0.235	1.15	1.677	12.51
88.49	0.219	1.20	1.724	11.51
89.44	0.204	1.25	1.772	10.56
90.32	0.189	1.30	1.821	9.63
91.15	0.175	1.35	1.872	8.35
91.92	0.162	1.40	1.923	8.08
92.65	0.150	1.45	1.977	7.35
93.32	0.133	1.50	2.033	6.68
93.94	0.127	1.55	2.098	6.06
94.52	0.117	1.60	2.147	5.48
95.05	0.107	1.65	2.208	4.95
95.34	0.098	1.70	2.270	4.46
95.99	0.090	1.75	2.335	4.01
96.41	0.082	1.80	2.403	3.59
96.78	0.074	1.85	2.473	3.22
97.13	0.067	1.90	2.546	2.87
97.44	0.061	1.95	2.622	2.56
97.72	0.055	2.00	2.701	2.28
97.98	0.050	2.05	2.784	2.02
98.21	0.045	2.10	2.870	1.79
98.42	0.040	2.15	2.961	1.58
98.61	0.036	2.20	3.055	1.39
98.78	0.032	2.25	3.155	1.22

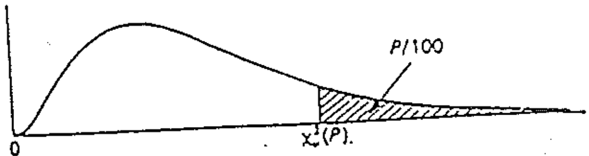
TABLE 8. PERCENTAGE POINTS OF THE χ^2-DISTRIBUTION

This table gives percentage points $\chi^2_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{2^{\nu/2}\Gamma(\frac{\nu}{2})}\int_{\chi^2_\nu(P)}^{\infty} x^{\frac{1}{2}\nu-1}e^{-\frac{1}{2}x}\,dx.$$

If X is a variable distributed as χ^2 with ν degrees of freedom, $P/100$ is the probability that $X \geqslant \chi^2_\nu(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu-1}$ and unit variance.



(The above shape applies for $\nu \geqslant 3$ only. When $\nu < 3$ the mode is at the origin.)

P	50	40	30	20	10	5	2·5	1	0·5	0·1	0·05
ν = 1	0·4549	0·7083	1·074	1·642	2·706	3·841	5·024	6·635	7·879	10·83	12·12
2	1·386	1·833	2·408	3·219	4·605	5·991	7·378	9·210	10·60	13·82	15·20
3	2·366	2·946	3·665	4·642	6·251	7·815	9·348	11·34	12·84	16·27	17·73
4	3·357	4·045	4·878	5·989	7·779	9·488	11·14	13·28	14·86	18·47	20·00
5	4·351	5·132	6·064	7·289	9·236	11·07	12·83	15·09	16·75	20·52	22·11
6	5·348	6·211	7·231	8·558	10·64	12·59	14·45	16·81	18·55	22·46	24·10
7	6·346	7·283	8·383	9·803	12·02	14·07	16·01	18·48	20·28	24·32	26·02
8	7·344	8·351	9·524	11·03	13·36	15·51	17·53	20·09	21·95	26·12	27·87
9	8·343	9·414	10·66	12·24	14·68	16·92	19·02	21·67	23·59	27·88	29·67
10	9·342	10·47	11·78	13·44	15·99	18·31	20·48	23·21	25·19	29·59	31·42
11	10·34	11·53	12·90	14·63	17·28	19·68	21·92	24·72	26·76	31·26	33·14
12	11·34	12·58	14·01	15·81	18·55	21·03	23·34	26·22	28·30	32·91	34·82
13	12·34	13·64	15·12	16·98	19·81	22·36	24·74	27·69	29·82	34·53	36·48
14	13·34	14·69	16·22	18·15	21·06	23·68	26·12	29·14	31·32	36·12	38·11
15	14·34	15·73	17·32	19·31	22·31	25·00	27·49	30·58	32·80	37·70	39·72
16	15·34	16·78	18·42	20·47	23·54	26·30	28·85	32·00	34·27	39·25	41·31
17	16·34	17·82	19·51	21·61	24·77	27·59	30·19	33·41	35·72	40·79	42·88
18	17·34	18·87	20·60	22·76	25·99	28·87	31·53	34·81	37·16	42·31	44·43
19	18·34	19·91	21·69	23·90	27·20	30·14	32·85	36·19	38·58	43·82	45·97
20	19·34	20·95	22·77	25·04	28·41	31·41	34·17	37·57	40·00	45·31	47·50
21	20·34	21·99	23·86	26·17	29·62	32·67	35·48	38·93	41·40	46·80	49·01
22	21·34	23·03	24·94	27·30	30·81	33·92	36·78	40·29	42·80	48·27	50·51
23	22·34	24·07	26·02	28·43	32·01	35·17	38·08	41·64	44·18	49·73	52·00
24	23·34	25·11	27·10	29·55	33·20	36·42	39·36	42·98	45·56	51·18	53·48
25	24·34	26·14	28·17	30·68	34·38	37·65	40·65	44·31	46·93	52·62	54·95
26	25·34	27·18	29·25	31·79	35·56	38·89	41·92	45·64	48·29	54·05	56·41
27	26·34	28·21	30·32	32·91	36·74	40·11	43·19	46·96	49·64	55·48	57·86
28	27·34	29·25	31·39	34·03	37·92	41·34	44·46	48·28	50·99	56·89	59·30
29	28·34	30·28	32·46	35·14	39·09	42·56	45·72	49·59	52·34	58·30	60·73
30	29·34	31·32	33·53	36·25	40·26	43·77	46·98	50·89	53·67	59·70	62·16
32	31·34	33·38	35·66	38·47	42·58	46·19	49·48	53·49	56·33	62·49	65·00
34	33·34	35·44	37·80	40·68	44·90	48·60	51·97	56·06	58·96	65·25	67·80
36	35·34	37·50	39·92	42·88	47·21	51·00	54·44	58·62	61·58	67·99	70·59
38	37·34	39·56	42·05	45·08	49·51	53·38	56·90	61·16	64·18	70·70	73·35
40	39·34	41·62	44·16	47·27	51·81	55·76	59·34	63·69	66·77	73·40	76·09
50	49·33	51·89	54·72	58·16	63·17	67·50	71·42	76·15	79·49	86·66	89·56
60	59·33	62·13	65·23	68·97	74·40	79·08	83·30	88·38	91·95	99·61	102·7
70	69·33	72·36	75·69	79·71	85·53	90·53	95·02	100·4	104·2	112·3	115·6
80	79·33	82·57	86·12	90·41	96·58	101·9	106·6	112·3	116·3	124·8	128·3
90	89·33	92·76	96·52	101·1	107·6	113·1	118·1	124·1	128·3	137·2	140·8
100	99·33	102·9	106·9	111·7	118·5	124·3	129·6	135·8	140·2	149·4	153·2

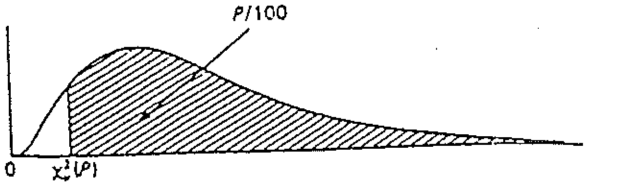
TABLE 8. PERÇENTAGE POINTS OF THE χ^2-DISTRIBUTION

This table gives percentage points $\chi^2_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{2^{\nu/2}\Gamma(\frac{\nu}{2})}\int_{\chi^2_\nu(P)}^{\infty} x^{\frac{1}{2}\nu-1}e^{-\frac{1}{2}x}\,dx.$$

If X is a variable distributed as χ^2 with ν degrees of freedom, $P/100$ is the probability that $X \geqslant \chi^2_\nu(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu-1}$ and unit variance.



(The above shape applies for $\nu \geqslant 3$ only. When $\nu < 3$ the mode is at the origin.)

P	99·95	99·9	99·5	99	97·5	95	90	80	70	60
$\nu = 1$	$0{\cdot}0^{6}3927$	$0{\cdot}0^{5}1571$	$0{\cdot}0^{4}3927$	$0{\cdot}0^{3}1571$	$0{\cdot}0^{3}9821$	0·003932	0·01579	0·06418	0·1485	0·2750
2	0·001000	0·002001	0·01003	0·02010	0·05064	0·1026	0·2107	0·4463	0·7133	1·022
3	0·01528	0·02430	0·07172	0·1148	0·2158	0·3518	0·5844	1·005	1·424	1·869
4	0·06392	0·09080	0·2070	0·2971	0·4844	0·7107	1·064	1·649	2·195	2·753
5	0·1581	0·2102	0·4117	0·5543	0·8312	1·145	1·610	2·343	3·000	3·655
6	0·2994	0·3811	0·6757	0·8721	1·237	1·635	2·204	3·070	3·828	4·570
7	0·4849	0·5985	0·9893	1·239	1·690	2·167	2·833	3·822	4·671	5·493
8	0·7104	0·8571	1·344	1·646	2·180	2·733	3·490	4·594	5·527	6·423
9	0·9717	1·152	1·735	2·088	2·700	3·325	4·168	5·380	6·393	7·357
10	1·265	1·479	2·156	2·558	3·247	3·940	4·865	6·179	7·267	8·295
11	1·587	1·834	2·603	3·053	3·816	4·575	5·578	6·989	8·148	9·237
12	1·934	2·214	3·074	3·571	4·404	5·226	6·304	7·807	9·034	10·18
13	2·305	2·617	3·565	4·107	5·009	5·892	7·042	8·634	9·926	11·13
14	2·697	3·041	4·075	4·660	5·629	6·571	7·790	9·467	10·82	12·08
15	3·108	3·483	4·601	5·229	6·262	7·261	8·547	10·31	11·72	13·03
16	3·536	3·942	5·142	5·812	6·908	7·962	9·312	11·15	12·62	13·98
17	3·980	4·416	5·697	6·408	7·564	8·672	10·09	12·00	13·53	14·94
18	4·439	4·905	6·265	7·015	8·231	9·390	10·86	12·86	14·44	15·89
19	4·912	5·407	6·844	7·633	8·907	10·12	11·65	13·72	15·35	16·85
20	5·398	5·921	7·434	8·260	9·591	10·85	12·44	14·58	16·27	17·81
21	5·896	6·447	8·034	8·897	10·28	11·59	13·24	15·44	17·18	18·77
22	6·404	6·983	8·643	9·542	10·98	12·34	14·04	16·31	18·10	19·73
23	6·924	7·529	9·260	10·20	11·69	13·09	14·85	17·19	19·02	20·69
24	7·453	8·085	9·886	10·86	12·40	13·85	15·66	18·06	19·94	21·65
25	7·991	8·649	10·52	11·52	13·12	14·61	16·47	18·94	20·87	22·62
26	8·538	9·222	11·16	12·20	13·84	15·38	17·29	19·82	21·79	23·58
27	9·093	9·803	11·81	12·88	14·57	16·15	18·11	20·70	22·72	24·54
28	9·656	10·39	12·46	13·56	15·31	16·93	18·94	21·59	23·65	25·51
29	10·23	10·99	13·12	14·26	16·05	17·71	19·77	22·48	24·58	26·48
30	10·80	11·59	13·79	14·95	16·79	18·49	20·60	23·36	25·51	27·44
32	11·98	12·81	15·13	16·36	18·29	20·07	22·27	25·15	27·37	29·38
34	13·18	14·06	16·50	17·79	19·81	21·66	23·95	26·94	29·24	31·31
36	14·40	15·32	17·89	19·23	21·34	23·27	25·64	28·73	31·12	33·25
38	15·64	16·61	19·29	20·69	22·88	24·88	27·34	30·54	32·99	35·19
40	16·91	17·92	20·71	22·16	24·43	26·51	29·05	32·34	34·87	37·13
50	23·46	24·67	27·99	29·71	32·36	34·76	37·69	41·45	44·31	46·86
60	30·34	31·74	35·53	37·48	40·48	43·19	46·46	50·64	53·81	56·62
70	37·47	39·04	43·28	45·44	48·76	51·74	55·33	59·90	63·35	66·40
80	44·79	46·52	51·17	53·54	57·15	60·39	64·28	69·21	72·92	76·19
90	52·28	54·16	59·20	61·75	65·65	69·13	73·29	78·56	82·51	85·99
100	59·90	61·92	67·33	70·06	74·22	77·93	82·36	87·95	92·13	95·81

TABLE 3

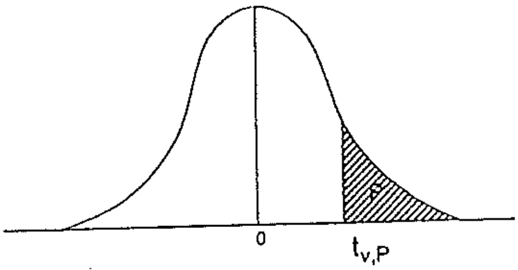
The t-Distribution :
Upper Probability Points

$P = P(t \geq t_{v,P}) = P(t \leq -t_{v,P})$

with $t_{v,P} = -t_{v,1-P}$ so that

$P(|t| \geq t_{v,P}) = 2P, \quad t_{v,P} > 0.$

Entries in the table are the values $t_{v,P}$ of the t-distribution for various degrees of freedom v and one tailed probabilities P.



v \ P	0.25	0.10	0.05	0.025	0.01	0.005
1	1.000	3.078	6.314	12.706	31.821	63.657
2	0.816	1.886	2.920	4.303	6.965	9.925
3	0.765	1.638	2.353	3.182	4.541	5.841
4	0.741	1.533	2.132	2.776	3.747	4.604
5	0.727	1.476	2.015	2.571	3.365	4.032
6	0.718	1.440	1.943	2.447	3.143	3.707
7	0.711	1.415	1.895	2.365	2.998	3.499
8	0.706	1.397	1.860	2.306	2.896	3.355
9	0.703	1.383	1.833	2.262	2.821	3.250
10	0.700	1.372	1.812	2.228	2.764	3.169
11	0.697	1.363	1.796	2.201	2.718	3.106
12	0.695	1.356	1.782	2.179	2.681	3.055
13	0.694	1.350	1.771	2.160	2.650	3.012
14	0.692	1.345	1.761	2.145	2.624	2.977
15	0.691	1.341	1.753	2.131	2.602	2.947
16	0.690	1.337	1.746	2.120	2.583	2.921
17	0.689	1.333	1.740	2.110	2.567	2.898
18	0.688	1.330	1.734	2.101	2.552	2.878
19	0.688	1.328	1.729	2.093	2.539	2.861
20	0.687	1.325	1.725	2.086	2.528	2.845
21	0.686	1.323	1.721	2.080	2.518	2.831
22	0.686	1.321	1.717	2.074	2.508	2.819
23	0.685	1.319	1.714	2.069	2.500	2.807
24	0.685	1.318	1.711	2.064	2.492	2.797
25	0.684	1.316	1.708	2.060	2.485	2.787
26	0.684	1.315	1.706	2.056	2.479	2.779
27	0.684	1.314	1.703	2.052	2.473	2.771
28	0.683	1.313	1.701	2.048	2.467	2.763
29	0.683	1.311	1.699	2.045	2.462	2.756
30	0.683	1.310	1.697	2.042	2.457	2.750
35	0.682	1.306	1.690	2.030	2.438	2.724
40	0.681	1.303	1.684	2.021	2.423	2.704
60	0.679	1.296	1.671	2.000	2.390	2.660
100	0.677	1.290	1.660	1.984	2.364	2.626
∞	0.675	1.282	1.645	1.960	2.326	2.576

SOME USEFUL FORMULAE

$$s^2 = \frac{1}{n-1}\sum(x-\bar{x})^2$$

$$\gamma(h) = C(\frac{3h}{2a} - \frac{h^3}{2a^3})$$

$$s^2 = \frac{1}{n-1}\sum x^2 - n\bar{x}^2$$

$$\gamma(h) = C(1-\exp(\frac{-h}{a}))$$

$$T = \frac{\bar{x}-\mu}{s\sqrt{n}}$$

$$T = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

$$P = PV\frac{i}{1-(1+i)^n}$$

$$PV = A/(1+r)^n$$

$$PV = P(1-(1+i)^n)/i$$

$$\theta = \frac{Mx^2 - x_p x_{1-p}}{x_p + x_{1-p} - 2Mx}$$

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

$$2\gamma(h) = \frac{1}{n}\sum\left[g(x) - g(x+h)\right]$$

$$r = \frac{\sum xy - \sum x \sum y/n}{[\sum x^2 - (\sum x)^2/n][\sum y^2 - (\sum y)^2/n]}$$

$$\sigma^e = \hat{s}_y\sqrt{1-r^2}$$