**REPUBLIC OF SOUTH-AFRICA**
**DEPARTMENT OF MINERALS AND ENERGY**
**EXAMINATION FOR THE MINE SURVEYOR'S CERTIFICATE OF COMPETENCY**

DATE: 14 October 2004 (Thursday)
TIME: 12:30 – 15:30 (3 Hours)

TOTAL MARKS: 100
TO PASS: 50

# MINING ECONOMICS II

NOTE:

1. Any pocket calculator may be used and intermediate results need not be shown. The make and model number of the calculator used must be noted on the front cover of answer book.
2. Tables that may be used are attached hereto.
3. Graph-, Probability-, and Log paper will be supplied if required.
4. Your examination number must be written on all graph paper and loose sheets that are handed in with your examination script.

## Question 1

a) What is the relationship between variance, semi-variance and co-variance? 2

b) Describe:
- i) Stationarity. 2
- ii) Intrinsic. 2
- iii) Isotropic. 1

c) Describe or sketch what you understand is meant by the following terms.

- i) Standard deviation 2
- ii) Negative skewness 2
- iii) Negative linear correlation 2
- iv) Kurtosis 2

[15 marks]

## Question 2

In the tabulation below, X represents the original estimated block values in g/t and Y the actual values of the blocks as they were mined.

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **X** | 6,1 | 6,2 | 6,4 | 6,4 | 6,6 | 6,8 | 6,9 | 6,9 | 7,2 | 7,4 | 7,6 | 7,7 | 7,7 | 7,8 |
| **Y** | 6,2 | 6,8 | 6,6 | 7,0 | 6,2 | 7,7 | 6,6 | 7,2 | 8,0 | 6,9 | 7,7 | 8,2 | 7,3 | 6,7 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **X** | 8,0 | 8,1 | 8,2 | 8,3 | 8,4 | 8,5 | 8,8 | 8,8 | 8,9 | 10,2 | 10,4 |
| **Y** | 8,0 | 8,6 | 7,4 | 7,0 | 8,2 | 7,8 | 8,7 | 8,1 | 8,5 | 8,4 | 9,0 |

Calculate the:

(a) Correlation coefficient.
(b) Regression line.
(c) Covariance.
(d) Test whether $\rho=0$ at the 0,05 level of significance.
(e) What is the probability distribution of the actual value of a block whose estimated value is 8,0g/t.
(f) Calculate the effective pay limit by which a sample may be judged if the mining pay limit is 7,0g/t.

[20 Marks]

## Question 3

Blocking of ore reserves at a certain mine (70m x 70m blocks) resulted in a mean block grade of 10,35g/t with a standard deviation of 2,96g/t for the 200 blocks.

a) What percentage of ore will be above the pay limit of 8,0g/t if the block size is reduced to 35m by 70m?

b) What will the value of the ore above pay limit be for the 35m x 70m blocks?

c) Calculate 90% confidence limits for the mean of the 70m x 70m blocks.

[10 Marks]

## Question 4

Ore blocks on a copper mine are normally distributed with a mean grade of 1,2% and variance 0,25% and adds up to 12,5 million tons.
Draw grade tonnage curves from 0,6% to 1,5% copper and measure from the graph what the pay tons and pay value at a cut off grade of 1,0%Cu is.

[20 Marks]

**Question 5**

Nine boreholes were drilled in an area known to conform to a lognormal distribution with an additive constant of 120 cmg/t.

| Borehole | Width | G/t |
|---|---|---|
| 1 | 81 | 4,0 |
| 2 | 79 | 5,4 |
| 3 | 101 | 2,3 |
| 4 | 66 | 19,7 |
| 5 | 75 | 15,6 |
| 6 | 89 | 4,2 |
| 7 | 84 | 3,9 |
| 8 | 63 | 6,2 |
| 9 | 82 | 5,8 |

The relative variance for the area is 0,2

a) Calculate the mean value. 5

b) Calculate the pay value at the pay limit. 3

c) How much of the resource will be payable at a pay limit of 6,0 g/t? 2

[10Marks]

**Question 6**

The table below show semi-variances calculated at ten metre lag distances.

a) Plot the experimental semi-variogram on the graph paper provided.
b) Fit an approximate semi-variogram model to the data.
c) What model best fits the data?
d) Estimate nugget effect, sill and range.
e) How would you handle the extreme deviation of lag 11?
f) What is the most likely cause for the "W" shape from lag 3 to lag 11?

| Lag | Average distance | Pairs | Semi Variance |
|---|---|---|---|
| 1 | 7,5 | 333 | 0,16 |
| 2 | 15,3 | 913 | 0,22 |
| 3 | 24,7 | 1218 | 0,28 |
| 4 | 34,9 | 1566 | 0,27 |
| 5 | 45,0 | 1630 | 0,24 |
| 6 | 54,8 | 1548 | 0,25 |
| 7 | 64,7 | 1292 | 0,28 |
| 8 | 74,9 | 1192 | 0,25 |
| 9 | 84,9 | 1041 | 0,23 |
| 10 | 94,8 | 315 | 0,26 |
| 11 | 104,7 | 241 | 0,33 |

[10Marks]

**Question 7**

By spending R21 million capital to upgrade the recovery plant during the current financial year, total throughput can be increased by 4 000 tons per month and gold recovery can be increased from 95,6% to 97,3%. If the current milling rate is 50 000t/m @ 8,7g/t and the gold price is expected to increase by 8,0% from the current R84 000/Kg, calculate the present value of the increase in profit over four years at an interest rate of 9,5%.

Will the capital be recovered after the four years?

[15 Marks]

[TOTAL 100 Marks]

SOME USEFUL FORMULAE

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N} (x_i - u)^2$$

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N} x_i^{\,2} - u^2$$

$$s^2 = \frac{1}{n-1}\sum (x - \bar{x})^2$$

$$\gamma(h) = C\left(\frac{3h}{2a} - \frac{h^3}{2a^3}\right)$$

$$s^2 = \frac{1}{n-1}\left[\sum x^2 - n\bar{x}^2\right]$$

$$\gamma(h) = C\left(1 - \exp\left(\frac{-h}{a}\right)\right)$$

$$T = \frac{\bar{x} - \mu}{s\sqrt{n}}$$

$$T = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

$$P = PV\frac{i}{1-(1+i)^n}$$

$$PV = A/(1+r)^n$$

$$PV = P(1-(1+i)^n)/i$$

$$PV = \frac{P[(1+r)^n - 1]}{r(1+r)^n}$$

$$\theta = \frac{Mx^2 - x_p x_{1-p}}{x_p + x_{1-p} - 2Mx}$$

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

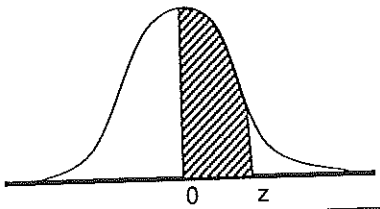
$$2\gamma(h) = \frac{1}{n}\sum \left[g(x) - g(x+h)\right]$$

$$r = \frac{\sum xy - \sum x \sum y / n}{\left[\sum x^2 - (\sum x)^2 / n\right]\left[\sum y^2 - (\sum y)^2 / n\right]}$$

$$\sigma^c = \hat{s}_y\sqrt{1-r^2}$$

TABLE 1

AREAS under the STANDARD NORMAL CURVE from 0 to z




| z | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0754 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1803 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2258 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2518 | 0.2549 |
| 0.7 | 0.2580 | 0.2612 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2996 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |
| 3.1 | 0.4990 | 0.4991 | 0.4991 | 0.4991 | 0.4992 | 0.4992 | 0.4992 | 0.4992 | 0.4993 | 0.4993 |
| 3.2 | 0.4993 | 0.4993 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4995 | 0.4995 | 0.4995 |
| 3.3 | 0.4995 | 0.4995 | 0.4995 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4997 |
| 3.4 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4998 |
| 3.5 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 |
| 3.6 | 0.4998 | 0.4998 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.7 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.8 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.9 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 |

TABLE 5

TABLE FOR THE COMPUTATION OF TONNAGE AND GRADE CUT-OFF FOR THE NORMAL DISTRIBUTION

| Cut-off below the mean | | Reduced cut-off = Z PV=MV+ ωσ | Cut-off above the mean | |
|---|---|---|---|---|
| Tonnage proportion | ω - factor | | ω - factor | Tonnage proportion |
| 50.00 | 0.798 | 0.00 | 0.798 | 50.00 |
| 51.99 | 0.766 | 0.05 | 0.830 | 48.01 |
| 53.98 | 0.735 | 0.10 | 0.863 | 46.02 |
| 55.96 | 0.705 | 0.15 | 0.896 | 44.04 |
| 57.93 | 0.675 | 0.20 | 0.929 | 42.07 |
| 59.87 | 0.645 | 0.25 | 0.964 | 40.13 |
| 61.79 | 0.617 | 0.30 | 0.998 | 38.21 |
| 63.68 | 0.589 | 0.35 | 1.034 | 36.32 |
| 65.54 | 0.562 | 0.40 | 1.069 | 34.45 |
| 67.36 | 0.535 | 0.45 | 1.106 | 32.64 |
| 69.15 | 0.509 | 0.50 | 1.142 | 30.85 |
| 70.88 | 0.484 | 0.55 | 1.180 | 29.12 |
| 72.57 | 0.459 | 0.60 | 1.217 | 27.43 |
| 74.22 | 0.435 | 0.65 | 1.256 | 25.78 |
| 75.80 | 0.411 | 0.70 | 1.295 | 24.20 |
| 77.34 | 0.389 | 0.75 | 1.334 | 22.66 |
| 78.81 | 0.367 | 0.80 | 1.375 | 21.19 |
| 80.23 | 0.346 | 0.85 | 1.415 | 19.77 |
| 81.59 | 0.326 | 0.90 | 1.457 | 18.41 |
| 82.89 | 0.306 | 0.95 | 1.499 | 17.11 |
| 84.13 | 0.287 | 1.00 | 1.542 | 15.87 |
| 85.31 | 0.269 | 1.05 | 1.586 | 14.69 |
| 86.43 | 0.251 | 1.10 | 1.631 | 13.57 |
| 87.49 | 0.235 | 1.15 | 1.677 | 12.51 |
| 88.49 | 0.219 | 1.20 | 1.724 | 11.51 |
| 89.44 | 0.204 | 1.25 | 1.772 | 10.56 |
| 90.32 | 0.189 | 1.30 | 1.821 | 9.63 |
| 91.15 | 0.175 | 1.35 | 1.872 | 8.35 |
| 91.92 | 0.162 | 1.40 | 1.923 | 8.08 |
| 92.65 | 0.150 | 1.45 | 1.977 | 7.35 |
| 93.32 | 0.133 | 1.50 | 2.033 | 6.68 |
| 93.94 | 0.127 | 1.55 | 2.098 | 6.06 |
| 94.52 | 0.117 | 1.60 | 2.147 | 5.48 |
| 95.05 | 0.107 | 1.65 | 2.208 | 4.95 |
| 95.34 | 0.098 | 1.70 | 2.270 | 4.46 |
| 95.99 | 0.090 | 1.75 | 2.335 | 4.01 |
| 96.41 | 0.082 | 1.80 | 2.403 | 3.59 |
| 96.78 | 0.074 | 1.85 | 2.473 | 3.22 |
| 97.13 | 0.067 | 1.90 | 2.546 | 2.87 |
| 97.44 | 0.061 | 1.95 | 2.622 | 2.56 |
| 97.72 | 0.055 | 2.00 | 2.701 | 2.28 |
| 97.98 | 0.050 | 2.05 | 2.784 | 2.02 |
| 98.21 | 0.045 | 2.10 | 2.870 | 1.79 |
| 98.42 | 0.040 | 2.15 | 2.961 | 1.58 |
| 98.61 | 0.036 | 2.20 | 3.055 | 1.39 |
| 98.78 | 0.032 | 2.25 | 3.155 | 1.22 |

LOWER 95%

Factor $b_{0.05}$ (v;n) for estimation of one sided lower 95% confidence limits of the mean of a lognormal population

| v \ n | 5 | 10 | 15 | 20 | 50 | 100 | 1000 |
|---|---|---|---|---|---|---|---|
| 0.00 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 0.02 | 0.8978 | 0.9333 | 0.9458 | 0.9540 | 0.9697 | 0.9782 | 0.9927 |
| 0.04 | 0.8589 | 0.9071 | 0.9246 | 0.9344 | 0.9573 | 0.9692 | 0.9895 |
| 0.06 | 0.8302 | 0.8874 | 0.9079 | 0.9200 | 0.9478 | 0.9622 | 0.9872 |
| 0.08 | 0.8070 | 0.8708 | 0.8943 | 0.9077 | 0.9398 | 0.9564 | 0.9852 |
| 0.10 | 0.7870 | 0.8563 | 0.8821 | 0.8972 | 0.9328 | 0.9512 | 0.9833 |
| 0.12 | 0.7693 | 0.8439 | 0.8716 | 0.8878 | 0.9264 | 0.9464 | 0.9817 |
| 0.14 | 0.7535 | 0.8323 | 0.8617 | 0.8790 | 0.9204 | 0.9420 | 0.9801 |
| 0.16 | 0.7389 | 0.8216 | 0.8527 | 0.8709 | 0.9149 | 0.9380 | 0.9787 |
| 0.18 | 0.7255 | 0.8116 | 0.8442 | 0.8632 | 0.9097 | 0.9341 | 0.9773 |
| 0.20 | 0.7129 | 0.8023 | 0.8360 | 0.8558 | 0.9048 | 0.9304 | 0.9760 |
| 0.30 | 0.6605 | 0.7618 | 0.8008 | 0.8243 | 0.8828 | 0.9139 | 0.9701 |
| 0.40 | 0.6187 | 0.7284 | 0.7717 | 0.7981 | 0.8639 | 0.8996 | 0.9648 |
| 0.50 | 0.5838 | 0.6995 | 0.7462 | 0.7744 | 0.8470 | 0.8867 | 0.9600 |
| 0.60 | 0.5538 | 0.6739 | 0.7270 | 0.7534 | 0.8313 | 0.8741 | 0.9554 |
| 0.70 | 0.5277 | 0.6508 | 0.7020 | 0.7338 | 0.8168 | 0.8632 | 0.9511 |
| 0.80 | 0.5044 | 0.6297 | 0.6825 | 0.7156 | 0.8030 | 0.8525 | 0.9470 |
| 0.90 | 0.4836 | 0.6103 | 0.6646 | 0.6987 | 0.7899 | 0.8421 | 0.9429 |
| 1.00 | 0.4650 | 0.5923 | 0.6476 | 0.6826 | 0.7774 | 0.8322 | 0.9389 |
| 1.10 | 0.4481 | 0.5756 | 0.6317 | 0.6674 | 0.7654 | 0.8226 | 0.9351 |
| 1.20 | 0.4328 | 0.5599 | 0.6165 | 0.6530 | 0.7538 | 0.8133 | 0.9313 |
| 1.30 | 0.4189 | 0.5452 | 0.6023 | 0.6393 | 0.7426 | 0.8042 | 0.9276 |
| 1.40 | 0.4062 | 0.5315 | 0.5888 | 0.6262 | 0.7318 | 0.7954 | 0.9240 |
| 1.50 | 0.3946 | 0.5186 | 0.5760 | 0.6137 | 0.7214 | 0.7868 | 0.9203 |
| 1.60 | 0.3840 | 0.5065 | 0.5637 | 0.6018 | 0.7112 | 0.7784 | 0.9168 |
| 1.70 | 0.3743 | 0.4950 | 0.5521 | 0.5904 | 0.7014 | 0.7702 | 0.9133 |
| 1.80 | 0.3655 | 0.4842 | 0.5410 | 0.5794 | 0.6918 | 0.7622 | 0.9098 |
| 1.90 | 0.3574 | 0.4740 | 0.5305 | 0.5688 | 0.6825 | 0.7544 | 0.9064 |
| 2.00 | 0.3501 | 0.4644 | 0.5203 | 0.5587 | 0.6734 | 0.7466 | 0.9030 |
| 2.10 | 0.3433 | 0.4552 | 0.5106 | 0.5489 | 0.6646 | 0.7391 | 0.8996 |
| 2.20 | 0.3372 | 0.4466 | 0.5014 | 0.5395 | 0.6560 | 0.7317 | 0.8962 |
| 2.30 | 0.3316 | 0.4385 | 0.4925 | 0.5304 | 0.6476 | 0.7245 | 0.8929 |
| 2.40 | 0.3266 | 0.4308 | 0.4840 | 0.5217 | 0.6394 | 0.7173 | 0.8896 |
| 2.50 | 0.3220 | 0.4234 | 0.4759 | 0.5133 | 0.6314 | 0.7104 | 0.8864 |
| 2.60 | 0.3179 | 0.4166 | 0.4681 | 0.5044 | 0.6236 | 0.7035 | 0.8831 |
| 2.70 | 0.3142 | 0.4100 | 0.4606 | 0.4974 | 0.6160 | 0.6967 | 0.8799 |
| 2.80 | 0.3110 | 0.4039 | 0.4535 | 0.4899 | 0.6085 | 0.6901 | 0.8767 |
| 2.90 | 0.3081 | 0.3981 | 0.4467 | 0.4826 | 0.6012 | 0.6836 | 0.8736 |
| 3.00 | 0.3055 | 0.3926 | 0.4401 | 0.4756 | 0.5941 | 0.6772 | 0.8704 |

UPPER 95%

Factor $b_{0.95}$ (v;n) for estimation of one sided upper 95% confidence limits of the mean of a lognormal population

| v \ n | 5 | 10 | 15 | 20 | 50 | 100 | 1000 |
|---|---|---|---|---|---|---|---|
| 0.00 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.02 | 1.241 | 1.117 | 1.084 | 1.067 | 1.038 | 1.026 | 1.007 |
| 0.04 | 1.362 | 1.171 | 1.122 | 1.099 | 1.055 | 1.037 | 1.011 |
| 0.06 | 1.466 | 1.216 | 1.154 | 1.124 | 1.069 | 1.046 | 1.013 |
| 0.08 | 1.561 | 1.256 | 1.181 | 1.146 | 1.080 | 1.053 | 1.015 |
| 0.10 | 1.652 | 1.293 | 1.207 | 1.166 | 1.091 | 1.060 | 1.017 |
| 0.12 | 1.740 | 1.327 | 1.230 | 1.184 | 1.100 | 1.066 | 1.019 |
| 0.14 | 1.827 | 1.361 | 1.253 | 1.202 | 1.109 | 1.072 | 1.020 |
| 0.16 | 1.914 | 1.393 | 1.274 | 1.219 | 1.118 | 1.078 | 1.022 |
| 0.18 | 1.999 | 1.425 | 1.295 | 1.236 | 1.126 | 1.084 | 1.023 |
| 0.20 | 2.087 | 1.455 | 1.316 | 1.252 | 1.135 | 1.089 | 1.025 |
| 0.30 | 2.532 | 1.606 | 1.415 | 1.328 | 1.172 | 1.113 | 1.031 |
| 0.40 | 3.019 | 1.756 | 1.509 | 1.399 | 1.207 | 1.135 | 1.037 |
| 0.50 | 3.563 | 1.910 | 1.603 | 1.470 | 1.240 | 1.156 | 1.042 |
| 0.60 | 4.176 | 2.070 | 1.682 | 1.541 | 1.273 | 1.175 | 1.047 |
| 0.70 | 4.870 | 2.237 | 1.798 | 1.614 | 1.306 | 1.196 | 1.052 |
| 0.80 | 5.663 | 2.415 | 1.901 | 1.688 | 1.338 | 1.215 | 1.057 |
| 0.90 | 6.570 | 2.604 | 2.006 | 1.763 | 1.371 | 1.235 | 1.062 |
| 1.00 | 7.605 | 2.805 | 2.117 | 1.842 | 1.404 | 1.254 | 1.067 |
| 1.10 | 8.795 | 3.019 | 2.233 | 1.924 | 1.437 | 1.274 | 1.071 |
| 1.20 | 10.155 | 3.250 | 2.355 | 2.008 | 1.471 | 1.294 | 1.076 |
| 1.30 | 11.718 | 3.497 | 2.483 | 2.096 | 1.506 | 1.314 | 1.080 |
| 1.40 | 13.513 | 3.761 | 2.617 | 2.187 | 1.540 | 1.334 | 1.085 |
| 1.50 | 15.569 | 4.045 | 2.758 | 2.282 | 1.576 | 1.354 | 1.089 |
| 1.60 | 17.928 | 4.351 | 2.907 | 2.380 | 1.613 | 1.374 | 1.094 |
| 1.70 | 20.639 | 4.680 | 3.064 | 2.484 | 1.650 | 1.395 | 1.098 |
| 1.80 | 23.749 | 5.034 | 3.229 | 2.592 | 1.688 | 1.416 | 1.103 |
| 1.90 | 27.318 | 5.414 | 3.403 | 2.704 | 1.728 | 1.438 | 1.107 |
| 2.00 | 31.398 | 5.825 | 3.588 | 2.822 | 1.767 | 1.459 | 1.112 |
| 2.10 | 36.079 | 6.268 | 3.783 | 2.945 | 1.808 | 1.481 | 1.116 |
| 2.20 | 41.444 | 6.745 | 3.989 | 3.074 | 1.850 | 1.504 | 1.121 |
| 2.30 | 47.586 | 7.260 | 4.208 | 3.209 | 1.893 | 1.526 | 1.125 |
| 2.40 | 54.611 | 7.815 | 4.438 | 3.351 | 1.937 | 1.549 | 1.130 |
| 2.50 | 62.661 | 8.415 | 4.683 | 3.498 | 1.982 | 1.572 | 1.134 |
| 2.60 | 71.861 | 9.061 | 4.941 | 3.670 | 2.029 | 1.596 | 1.139 |
| 2.70 | 82.366 | 9.759 | 5.214 | 3.816 | 2.076 | 1.620 | 1.144 |
| 2.80 | 94.377 | 10.512 | 5.504 | 3.986 | 2.125 | 1.645 | 1.148 |
| 2.90 | 108.115 | 11.326 | 5.811 | 4.164 | 2.175 | 1.670 | 1.153 |
| 3.00 | 123.750 | 12.206 | 6.137 | 4.351 | 2.226 | 1.695 | 1.158 |

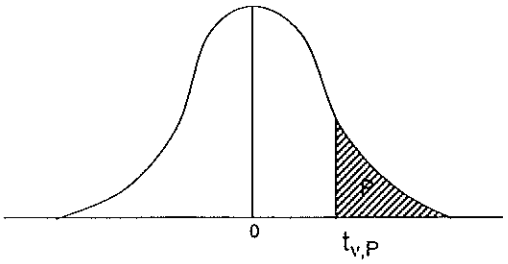
# FACTOR $\Upsilon_\eta(\nu)$ FOR ESTIMATION OF MEAN OF LOGNORMAL POPULATION

| $\nu$ \ $\eta$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.02 | 1.010 | 1.010 | 1.010 | 1.010 | 1.010 | 1.010 | 1.010 | 1.010 | 1.010 | 1.010 | 1.010 |
| 0.04 | 1.020 | 1.020 | 1.020 | 1.020 | 1.020 | 1.020 | 1.020 | 1.020 | 1.020 | 1.020 | 1.020 |
| 0.06 | 1.030 | 1.030 | 1.030 | 1.030 | 1.030 | 1.030 | 1.030 | 1.030 | 1.030 | 1.030 | 1.030 |
| 0.08 | 1.040 | 1.040 | 1.040 | 1.040 | 1.040 | 1.041 | 1.041 | 1.041 | 1.041 | 1.041 | 1.041 |
| 0.10 | 1.050 | 1.051 | 1.051 | 1.051 | 1.051 | 1.051 | 1.051 | 1.051 | 1.051 | 1.051 | 1.051 |
| 0.12 | 1.061 | 1.061 | 1.061 | 1.061 | 1.061 | 1.061 | 1.061 | 1.061 | 1.061 | 1.062 | 1.062 |
| 0.14 | 1.071 | 1.071 | 1.071 | 1.072 | 1.072 | 1.072 | 1.072 | 1.072 | 1.072 | 1.072 | 1.072 |
| 0.16 | 1.081 | 1.082 | 1.082 | 1.082 | 1.082 | 1.082 | 1.082 | 1.083 | 1.083 | 1.083 | 1.083 |
| 0.18 | 1.091 | 1.092 | 1.092 | 1.093 | 1.093 | 1.093 | 1.093 | 1.093 | 1.093 | 1.093 | 1.094 |
| 0.20 | 1.102 | 1.102 | 1.103 | 1.103 | 1.104 | 1.104 | 1.104 | 1.104 | 1.104 | 1.104 | 1.104 |
| 0.3 | 1.154 | 1.156 | 1.157 | 1.158 | 1.158 | 1.159 | 1.159 | 1.159 | 1.160 | 1.160 | 1.160 |
| 0.4 | 1.207 | 1.210 | 1.212 | 1.214 | 1.215 | 1.216 | 1.216 | 1.217 | 1.217 | 1.217 | 1.218 |
| 0.5 | 1.260 | 1.266 | 1.269 | 1.272 | 1.273 | 1.275 | 1.276 | 1.276 | 1.277 | 1.278 | 1.278 |
| 0.6 | 1.315 | 1.323 | 1.328 | 1.332 | 1.334 | 1.336 | 1.337 | 1.338 | 1.339 | 1.340 | 1.341 |
| 0.7 | 1.371 | 1.382 | 1.389 | 1.393 | 1.397 | 1.399 | 1.401 | 1.403 | 1.404 | 1.406 | 1.406 |
| 0.8 | 1.427 | 1.442 | 1.451 | 1.457 | 1.462 | 1.465 | 1.468 | 1.470 | 1.472 | 1.473 | 1.475 |
| 0.9 | 1.485 | 1.503 | 1.515 | 1.523 | 1.529 | 1.533 | 1.537 | 1.540 | 1.542 | 1.544 | 1.546 |
| 1.0 | 1.543 | 1.566 | 1.580 | 1.591 | 1.598 | 1.604 | 1.608 | 1.612 | 1.615 | 1.618 | 1.620 |
| 1.1 | 1.602 | 1.630 | 1.648 | 1.661 | 1.670 | 1.677 | 1.682 | 1.687 | 1.691 | 1.694 | 1.697 |
| 1.2 | 1.662 | 1.696 | 1.718 | 1.733 | 1.744 | 1.752 | 1.759 | 1.765 | 1.770 | 1.774 | 1.777 |
| 1.3 | 1.724 | 1.764 | 1.789 | 1.807 | 1.820 | 1.831 | 1.839 | 1.846 | 1.851 | 1.856 | 1.860 |
| 1.4 | 1.786 | 1.832 | 1.862 | 1.884 | 1.900 | 1.912 | 1.922 | 1.930 | 1.936 | 1.942 | 1.947 |
| 1.5 | 1.848 | 1.903 | 1.938 | 1.963 | 1.981 | 1.996 | 2.007 | 2.017 | 2.025 | 2.032 | 2.037 |
| 1.6 | 1.912 | 1.975 | 2.015 | 2.044 | 2.066 | 2.082 | 2.096 | 2.107 | 2.116 | 2.124 | 2.131 |
| 1.7 | 1.977 | 2.049 | 2.095 | 2.128 | 2.153 | 2.172 | 2.188 | 2.201 | 2.212 | 2.221 | 2.229 |
| 1.8 | 2.043 | 2.124 | 2.177 | 2.214 | 2.243 | 2.265 | 2.283 | 2.298 | 2.310 | 2.321 | 2.330 |
| 1.9 | 2.110 | 2.201 | 2.260 | 2.303 | 2.336 | 2.361 | 2.382 | 2.399 | 2.413 | 2.425 | 2.436 |
| 2.0 | 2.178 | 2.280 | 2.347 | 2.395 | 2.431 | 2.460 | 2.484 | 2.503 | 2.519 | 2.533 | 2.545 |
| 2.1 | 2.247 | 2.360 | 2.435 | 2.489 | 2.530 | 2.563 | 2.589 | 2.611 | 2.630 | 2.645 | 2.659 |
| 2.2 | 2.317 | 2.442 | 2.526 | 2.586 | 2.632 | 2.669 | 2.698 | 2.723 | 2.744 | 2.762 | 2.778 |
| 2.3 | 2.388 | 2.526 | 2.618 | 2.686 | 2.737 | 2.778 | 2.811 | 2.839 | 2.863 | 2.883 | 2.900 |
| 2.4 | 2.460 | 2.612 | 2.714 | 2.788 | 2.846 | 2.891 | 2.928 | 2.959 | 2.986 | 3.008 | 3.028 |
| 2.5 | 2.533 | 2.699 | 2.812 | 2.894 | 2.957 | 3.008 | 3.049 | 3.084 | 3.113 | 3.138 | 3.160 |
| 2.6 | 2.607 | 2.789 | 2.912 | 3.003 | 3.073 | 3.128 | 3.174 | 3.213 | 3.245 | 3.274 | 3.298 |
| 2.7 | 2.682 | 2.880 | 3.015 | 3.114 | 3.191 | 3.253 | 3.304 | 3.346 | 3.382 | 3.414 | 3.441 |
| 2.8 | 2.759 | 2.973 | 3.120 | 3.229 | 3.314 | 3.382 | 3.437 | 3.484 | 3.524 | 3.559 | 3.589 |
| 2.9 | 2.836 | 3.068 | 3.228 | 3.347 | 3.440 | 3.514 | 3.576 | 3.627 | 3.671 | 3.710 | 3.743 |
| 3.0 | 2.914 | 3.166 | 3.339 | 3.469 | 3.570 | 3.651 | 3.718 | 3.775 | 3.824 | 3.866 | 3.902 |
| 3.1 | 2.994 | 3.265 | 3.453 | 3.593 | 3.703 | 3.792 | 3.866 | 3.928 | 3.981 | 4.028 | 4.068 |
| 3.2 | 3.075 | 3.366 | 3.569 | 3.721 | 3.841 | 3.938 | 4.018 | 4.086 | 4.145 | 4.195 | 4.240 |
| 3.3 | 3.157 | 3.469 | 3.688 | 3.853 | 3.983 | 4.088 | 4.176 | 4.250 | 4.314 | 4.369 | 4.418 |
| 3.4 | 3.240 | 3.574 | 3.810 | 3.988 | 4.129 | 4.243 | 4.338 | 4.419 | 4.489 | 4.549 | 4.603 |
| 3.5 | 3.324 | 3.682 | 3.935 | 4.127 | 4.279 | 4.403 | 4.506 | 4.594 | 4.670 | 4.736 | 4.794 |
| 3.6 | 3.409 | 3.792 | 4.063 | 4.270 | 4.434 | 4.568 | 4.680 | 4.775 | 4.858 | 4.929 | 4.993 |
| 3.7 | 3.496 | 3.903 | 4.194 | 4.416 | 4.593 | 4.738 | 4.859 | 4.962 | 5.052 | 5.130 | 5.198 |
| 3.8 | 3.583 | 4.017 | 4.329 | 4.567 | 4.757 | 4.913 | 5.044 | 5.156 | 5.252 | 5.337 | 5.412 |
| 3.9 | 3.672 | 4.134 | 4.466 | 4.721 | 4.925 | 5.093 | 5.234 | 5.355 | 5.460 | 5.552 | 5.633 |
| 4.0 | 3.762 | 4.252 | 4.607 | 4.880 | 5.099 | 5.279 | 5.431 | 5.562 | 5.675 | 5.774 | 5.862 |
| 4.1 | 3.853 | 4.373 | 4.751 | 5.042 | 5.277 | 5.471 | 5.634 | 5.775 | 5.897 | 6.004 | 6.099 |
| 4.2 | 3.946 | 4.496 | 4.898 | 5.209 | 5.460 | 5.668 | 5.844 | 5.995 | 6.127 | 6.242 | 6.345 |
| 4.3 | 4.040 | 4.622 | 5.049 | 5.380 | 5.649 | 5.872 | 6.060 | 6.223 | 6.364 | 6.489 | 6.599 |
| 4.4 | 4.135 | 4.750 | 5.203 | 5.556 | 5.843 | 6.081 | 6.283 | 6.458 | 6.610 | 6.744 | 6.863 |
| 4.5 | 4.231 | 4.881 | 5.361 | 5.736 | 6.042 | 6.297 | 6.513 | 6.700 | 6.863 | 7.008 | 7.136 |
| 4.6 | 4.328 | 5.014 | 5.522 | 5.921 | 6.247 | 6.519 | 6.750 | 6.950 | 7.126 | 7.281 | 7.419 |
| 4.7 | 4.427 | 5.149 | 5.687 | 6.111 | 6.457 | 6.747 | 6.995 | 7.209 | 7.397 | 7.563 | 7.711 |
| 4.8 | 4.527 | 5.288 | 5.856 | 6.305 | 6.674 | 6.983 | 7.247 | 7.476 | 7.677 | 7.855 | 8.014 |
| 4.9 | 4.629 | 5.428 | 6.029 | 6.505 | 6.896 | 7.225 | 7.507 | 7.751 | 7.966 | 8.157 | 8.328 |
| 5.0 | 4.732 | 5.572 | 6.205 | 6.709 | 7.124 | 7.474 | 7.774 | 8.036 | 8.265 | 8.470 | 8.652 |
| 5.1 | 4.836 | 5.718 | 6.386 | 6.919 | 7.359 | 7.731 | 8.050 | 8.329 | 8.574 | 8.792 | 8.988 |
| 5.2 | 4.941 | 5.866 | 6.570 | 7.134 | 7.600 | 7.995 | 8.335 | 8.631 | 8.893 | 9.126 | 9.335 |
| 5.3 | 5.048 | 6.018 | 6.759 | 7.354 | 7.847 | 8.266 | 8.628 | 8.944 | 9.222 | 9.471 | 9.695 |
| 5.4 | 5.156 | 6.172 | 6.951 | 7.579 | 8.102 | 8.546 | 8.930 | 9.265 | 9.563 | 9.828 | 10.07 |
| 5.5 | 5.266 | 6.329 | 7.148 | 7.811 | 8.363 | 8.833 | 9.240 | 9.598 | 9.914 | 10.20 | 10.45 |
| 5.6 | 5.376 | 6.489 | 7.350 | 8.048 | 8.631 | 9.129 | 9.561 | 9.940 | 10.28 | 10.58 | 10.85 |
| 5.7 | 5.489 | 6.652 | 7.555 | 8.290 | 8.906 | 9.433 | 9.890 | 10.29 | 10.65 | 10.97 | 11.26 |
| 5.8 | 5.603 | 6.818 | 7.766 | 8.539 | 9.188 | 9.745 | 10.23 | 10.66 | 11.04 | 11.38 | 11.68 |
| 5.9 | 5.718 | 6.987 | 7.980 | 8.794 | 9.478 | 10.07 | 10.58 | 11.03 | 11.44 | 11.80 | |
| 6.0 | 5.834 | 7.159 | 8.200 | 9.054 | 9.776 | 10.84 | 10.84 | 11.42 | 11.85 | | |

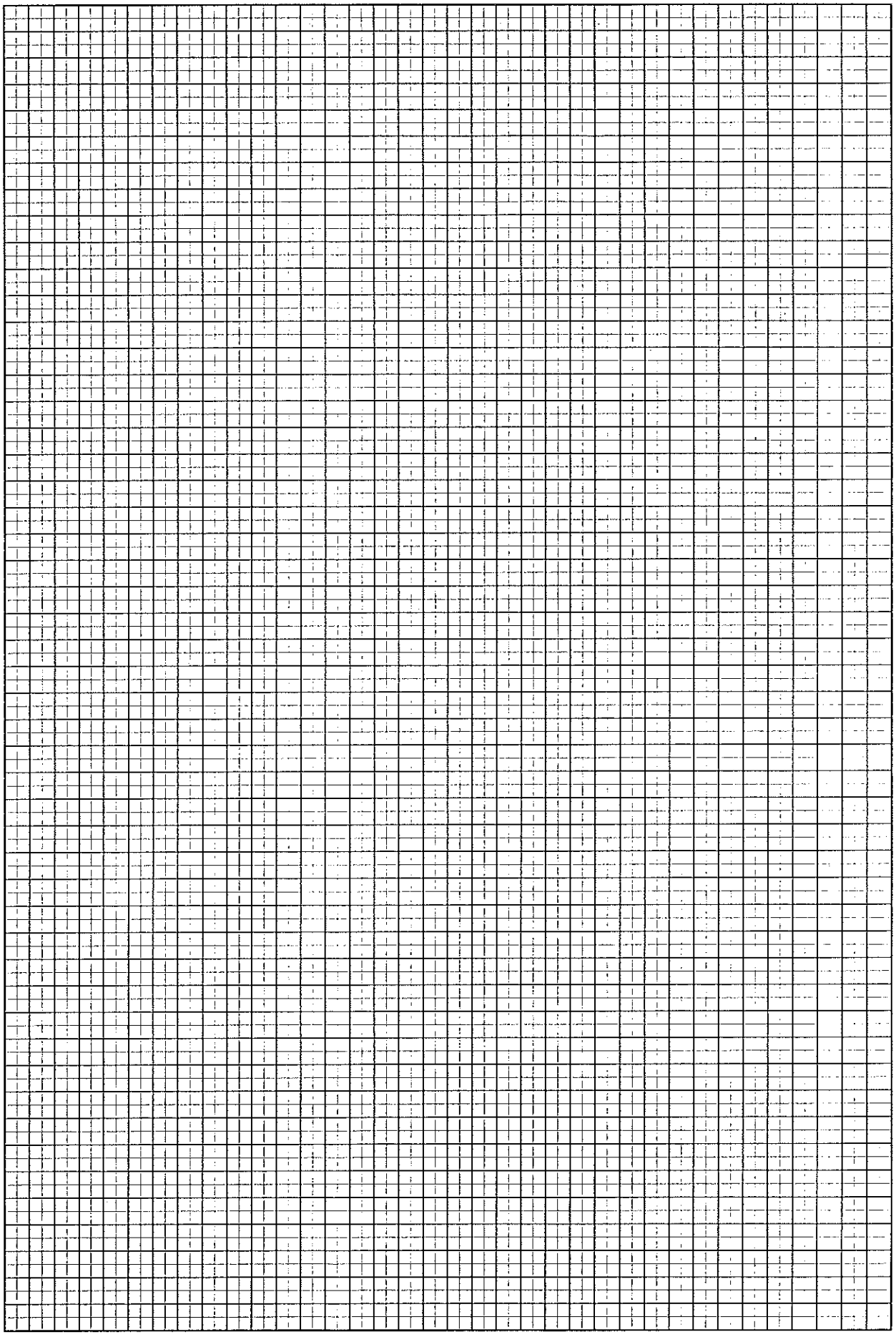
## TABLE 3

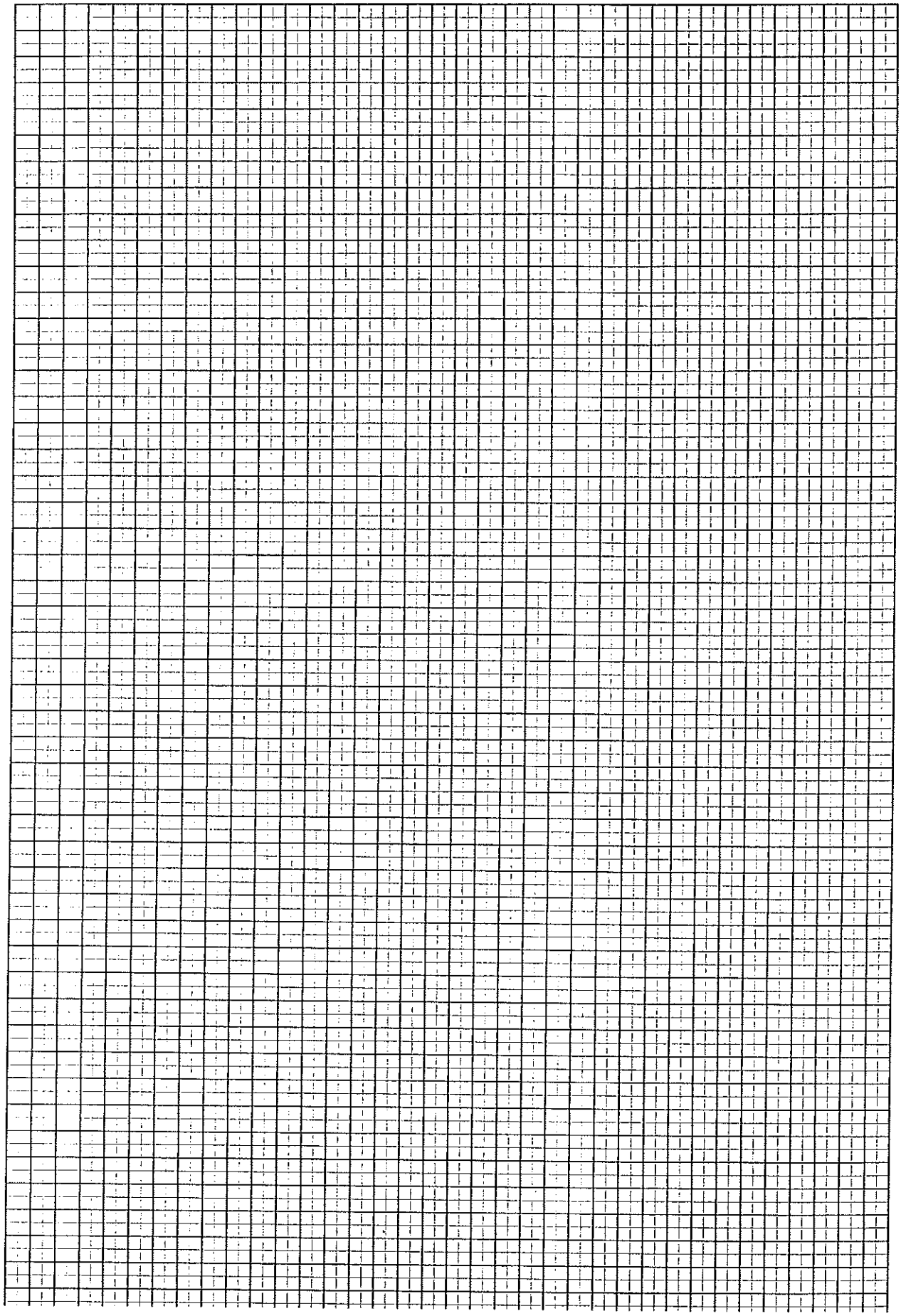
The t-Distribution :
Upper Probability Points
$P = P(t \geq t_{\nu,P}) = P(t \leq -t_{\nu,P})$
with $t_{\nu,P} = -t_{\nu,1-P}$ so that
$P(|t| \geq t_{\nu,P}) = 2P, \quad t_{\nu,P} > 0.$

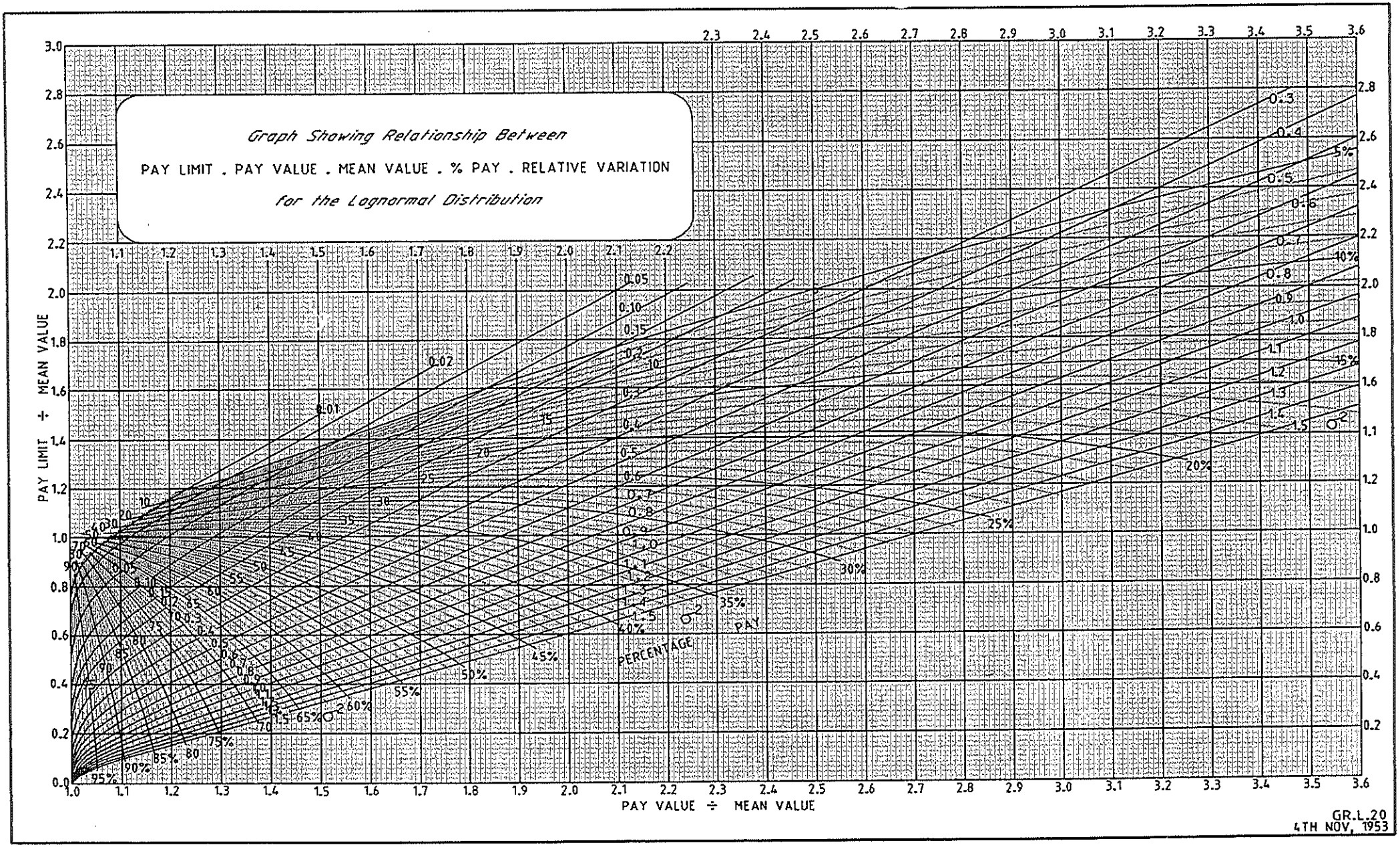
Entries in the table are the values $t_{\nu,P}$ of the t-distribution for various degrees of freedom $\nu$ and one tailed probabilities P.




| $\nu$ \ P | 0.25 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
|---|---|---|---|---|---|---|
| 1 | 1.000 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | 0.816 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | 0.765 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | 0.741 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | 0.727 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 |
| 6 | 0.718 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | 0.711 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | 0.706 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | 0.703 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | 0.700 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 |
| 11 | 0.697 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 |
| 12 | 0.695 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 |
| 13 | 0.694 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 |
| 14 | 0.692 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 |
| 15 | 0.691 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 |
| 16 | 0.690 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 |
| 17 | 0.689 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 |
| 18 | 0.688 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 |
| 19 | 0.688 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 |
| 20 | 0.687 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 |
| 21 | 0.686 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 |
| 22 | 0.686 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 |
| 23 | 0.685 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 |
| 24 | 0.685 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 |
| 25 | 0.684 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 |
| 26 | 0.684 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 |
| 27 | 0.684 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 |
| 28 | 0.683 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 |
| 29 | 0.683 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 |
| 30 | 0.683 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 |
| 35 | 0.682 | 1.306 | 1.690 | 2.030 | 2.438 | 2.724 |
| 40 | 0.681 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 |
| 60 | 0.679 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 |
| 100 | 0.677 | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 |
| $\infty$ | 0.675 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 |






Graph Showing Relationship Between
PAY LIMIT . PAY VALUE . MEAN VALUE . % PAY . RELATIVE VARIATION
for the Lognormal Distribution
PAY LIMIT ÷ MEAN VALUE
PAY VALUE ÷ MEAN VALUE
PERCENTAGE PAY
GR.L.20
4TH NOV, 1953